Conventional Nonlinear Gyrokinetic Equation

PPPL

[eg., Frieman and Chen, Phys. Fluids 1982]

- Foundations for Tokamak Nonlinear Kinetic Theory

  for analytic applications...

- Ordering is minimal,
  $\frac{\omega}{\Omega_i} \sim \frac{k_\parallel}{k_\perp} \sim \epsilon << 1, \frac{\delta f}{f_0} \sim \frac{e\delta\phi}{T_e} \sim \frac{1}{k_\perp L_p} \sim \epsilon << 1$

- and generic,
  $k_\perp \rho_i \sim 1 \rightarrow \omega \sim k_\parallel v_{Ti}$ for wave-particle resonance

- Based on direct gyro-phase average of Vlasov equation,
  Lots of algebra and book keeping.

- Energy, phase space volume **not** conserved.
  Velocity space nonlinearity (small) is ignored.
  $\rightarrow$ Long Term Behavior? [Villard, Hatzky, Sorge,...]

Phase Space Lagrangian Derivation of NL GK

[since Hahm, PF '88, followed by Brizard, Sugama,...]

Conservations Laws are Satisfied.

Various expansion parameters appear at different stages:

- Guiding center drift calculations in equilibrium field $\mathbf{B}$:
  Expansion in $\delta_B \equiv \rho_i / L_B$.

- Perturbative analysis consists of near-identity transformations to new variables which remove the gyro-phase dependence in perturbed fields $\delta \mathbf{A}(\mathbf{x}), \delta\phi(\mathbf{x})$ where $\mathbf{x} = \mathbf{R} + \rho$:
  Expansion in $\epsilon_\phi \equiv e(\delta\phi - \frac{v_\parallel}{c}\delta A_\parallel)/T_e \sim \delta B_\parallel / B_0$

- Hahm [PF 31, 2670 '88] argued that for tokamak core micro-turbulence, it is essential to keep $O(\epsilon_\phi^2) \sim O(\rho_*^2)$ for polarization shielding and energy conservation, while adequate to keep up to $O(\delta_B)$.

Nonlinear GK Equation for Tokamak Edge

- Perturbative analysis in fluctuation amplitude:

$$\gamma_1 = -e\delta\phi(\mathbf{R} + \rho)dt + e\delta\mathbf{A}(\mathbf{R} + \rho) \cdot (d\mathbf{R} + d\rho)$$

- Perform Lie-perturbation:

$$\Gamma_1 = \gamma_1 - L_1\gamma_0 + dS$$

- ...

- Gyrokinetic Maxwell's Equations
  via Pull-back Transformation

- Polarization shielding and energy conservation
  between particles and fields

Role of Turbulence Spreading in Edge-Core Coupling

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Ackn: K. Itoh (NIFS), M. Yagi, S.-I. Itoh (Kyushu U)

US-Japan Workshop on Theory-Based Modeling and
Integrated Simulation of Burning Plasma, Dec 15-17, '03, Kyoto, Japan

Outline

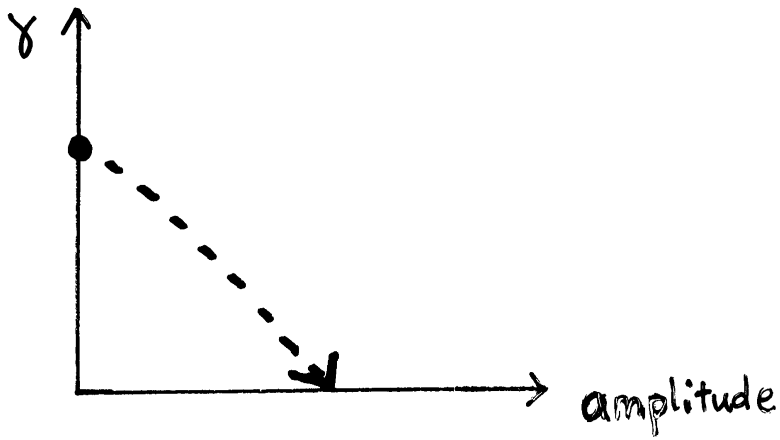
- Fluctuations in the Linearly Stable Zone: Observations from Gyrokinetic Simulations
- Characteristics of Tokamak Turbulence
- Spreading of Edge Turbulence into Interior
- Extended Nonlinear Gyrokinetic Equations for Dynamical Coupling near Pedestal

Determination of Fluctuation Amplitude

$$\gamma = \gamma_{lin} - k_\perp^2 D_{turb} \to 0$$

Nonlinear coupling induced dissipation leads to saturation

B. Kadomtsev '65



"Local Balance in Space"

"Conceptual Foundation of Most Transport Models"

Exception: Itoh-Itoh-Fukuyama-Yagi

Missing:

Meso-scale Dynamics: Barrier Movements, Avalanches,...

- **Non-zero Fluctuation Level in the Linearly Stable Zone**

Anomalous Transport where $\gamma_{lin} < 0$

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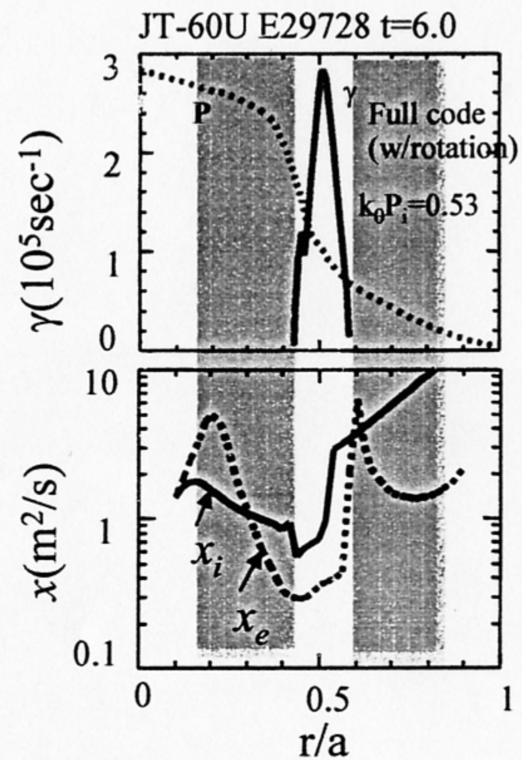
Core of Reversed Shear Plasmas
where profiles are nearly flat
(JT-60U, TFTR, DIII-D,...)

[*Rewoldt-Shirai, et al., NF, '02*]



→ Nonlinear Instability?

Self-sustained turbulence by B. Scott

Itoh et al.,'s work, ...

→ **Spreading from the Linearly Unstable Zone**

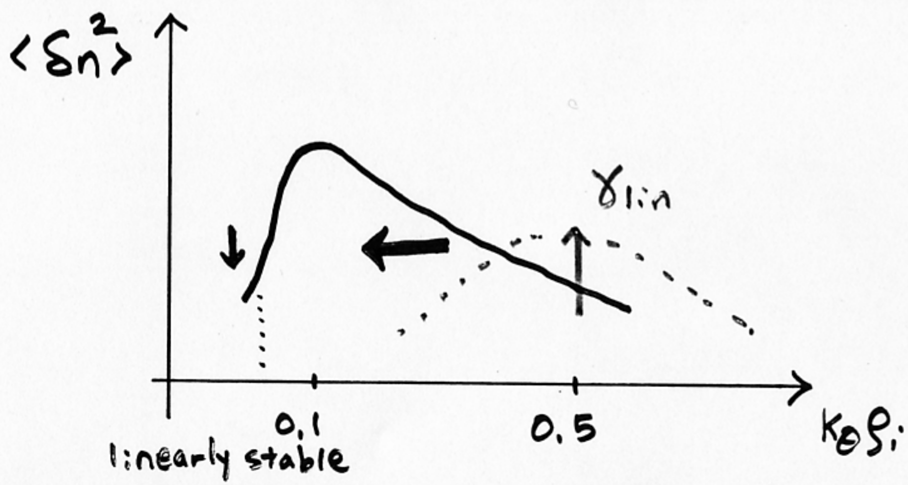
Excitation of Linearly Damped Modes

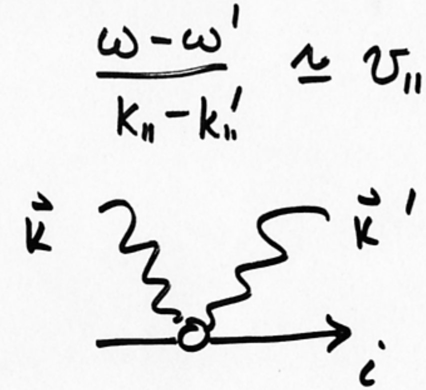
- Weak Turbulence Theory

  (Sagdeev and Galeev, *Nonlinear Plasma Theory* '69)

  Nonlinear Saturation from Balance between:

  $\underline{\gamma_{lin}}$ vs. Spectral Transfer from "Compton Scattering"





→ Non-zero Amplitude for Linearly Damped Modes in k-space

Gang-Diamond-Rosenbluth, PF-B 3, 68 (1991)

Hahm-Tang, PF-B 3, 989 (1991)

...

Horton, Rev. Mod .Phys (2000) *for more references.*

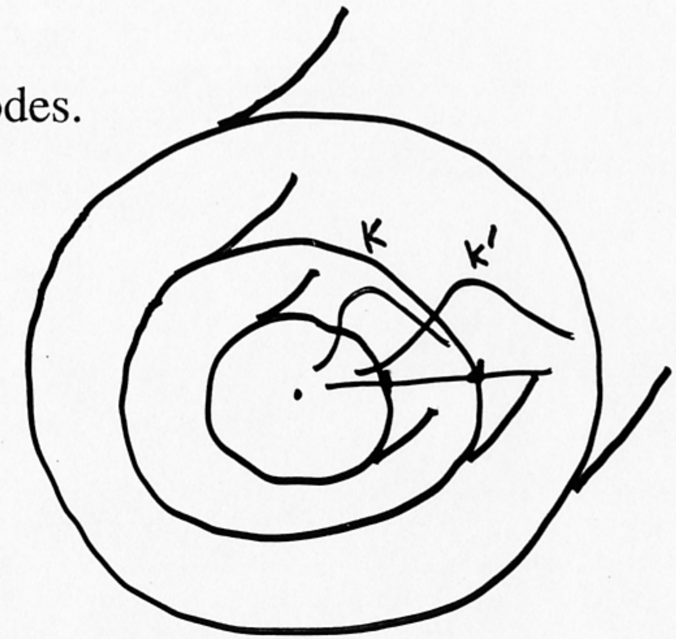
Nonlinear Coupling Leads To Radial Diffusion

Nonlinear interactions of modes: traditionally discussed in $\mathbf{k}$ space, but must spread fluctuation energy in radius due to:

i) $ik_x = \frac{\partial}{\partial x}$,

ii) mode location at rational surface,

iii) different radial extents of different modes.



$\mathbf{E} \times \mathbf{B}$ nonlinearity → "local turbulent damping" and "radial diffusion":

$$(\mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 R_{k,k'} I_k I_{k'} \rightarrow -\frac{\partial}{\partial x} D_r(I) \frac{\partial}{\partial x} I + k_\theta^2 D_\theta(I) I.$$

For details, see [eg., Kim, Diamond, Malkov, Hahm *et al.*, NF, 2003].

Theoretical Model of Turbulence Spreading

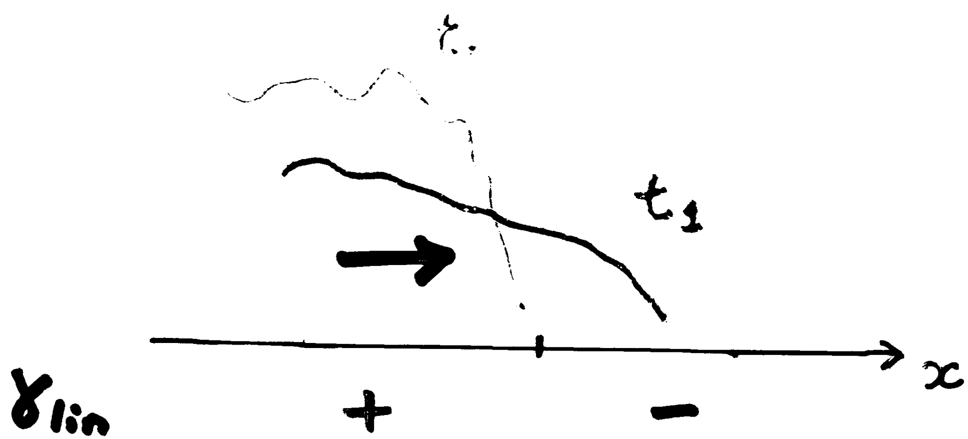
[**Hahm, Diamond, Lin, Itoh, Itoh**, *IAEA TM on H-mode, To appear in PPCF '03*]

$$\frac{\partial}{\partial t} I = \gamma(x) I - \alpha I^{1+\beta} + \chi_0 \frac{\partial}{\partial x}(I^\beta \frac{\partial}{\partial x} I)$$

I: turbulence intensity, $\gamma(x)$ is "local" growth rate,

α: a local nonlinear coupling, $\chi_0 I^\beta = \chi_i$ is a turbulent diffusivity

$\beta = 1 \rightarrow$ Weak Turbulence, $\beta = 1/2 \rightarrow$ Strong Turbulence

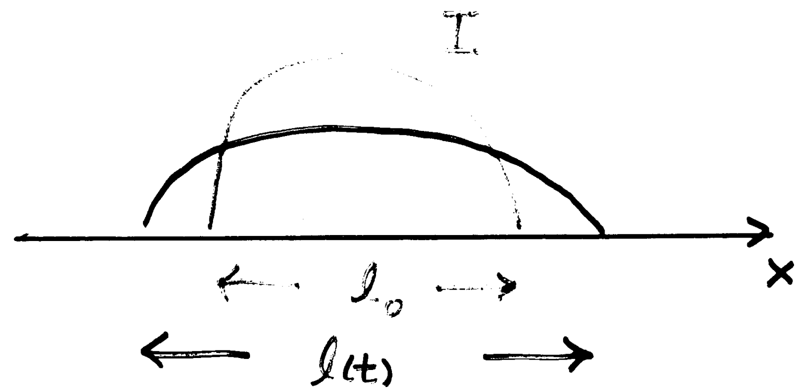


$$\frac{\partial}{\partial t} \int_{x-\Delta}^{x+\Delta} dx' I(x', t) \sim \chi_0 I^\beta \frac{\partial}{\partial x} I]_{x-\Delta}^{x+\Delta} + \ldots$$

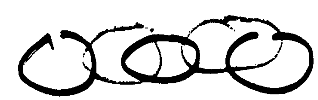
Profile of Fluctuation Intensity crucial to its Spatio-temporal Evolution

Long Term Behavior: Sub-Diffusion

- Self-similar Variable: $\ell(t)^2 \sim \chi_0 I^\beta t$
- $I(t)\ell(t) = I(0)\ell(0) \equiv \epsilon$, up to dissipation
- $\ell(t) \sim [\chi_0 \epsilon^\beta t]^{\frac{1}{2+\beta}}$
  $\sim t^{1/3}$: Weak Turbulence
  $\sim t^{2/5}$: Strong Turbulence



- Previous numerical mode coupling study:
  X. Garbet *et al.*, NF 1994



  Linear toroidal coupling usually dominates $\sim t^1$: convective
  Without linear toroidal mode coupling $\sim t^{1/2}$: **diffusive**

Dynamics of Turbulence Spreading

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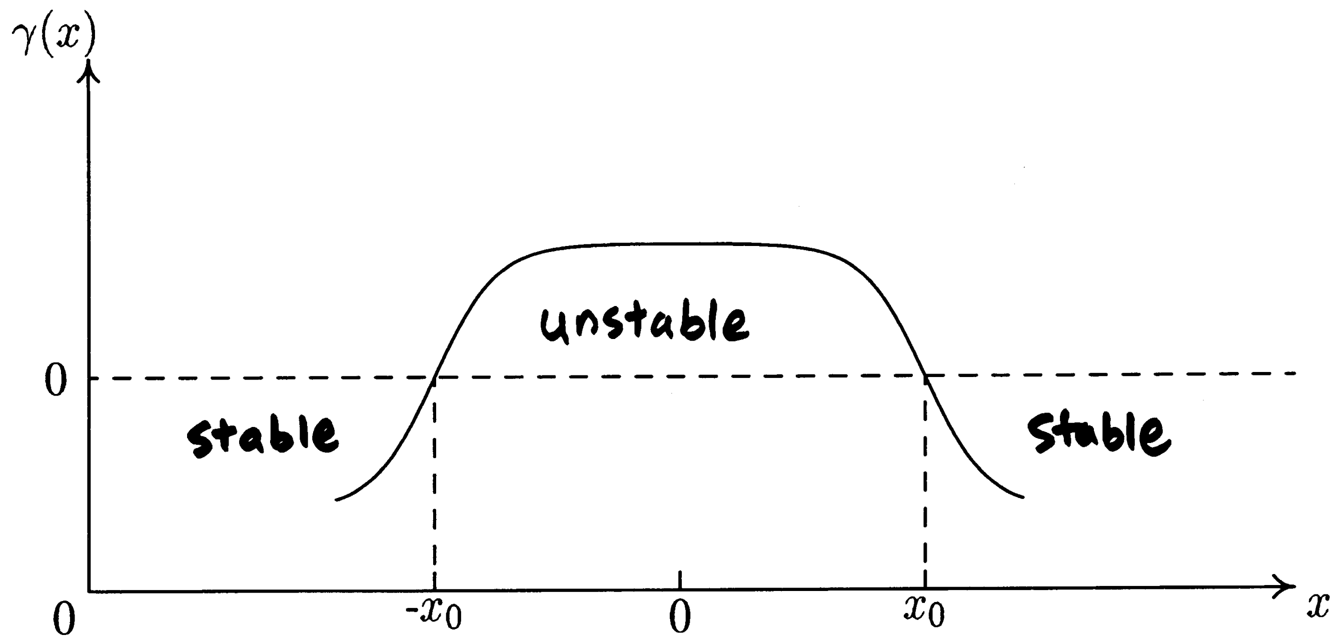
$$\frac{\partial}{\partial t}I = \gamma(x)I - \alpha I^2 + \chi_0\frac{\partial}{\partial x}(I\frac{\partial}{\partial x}I)$$

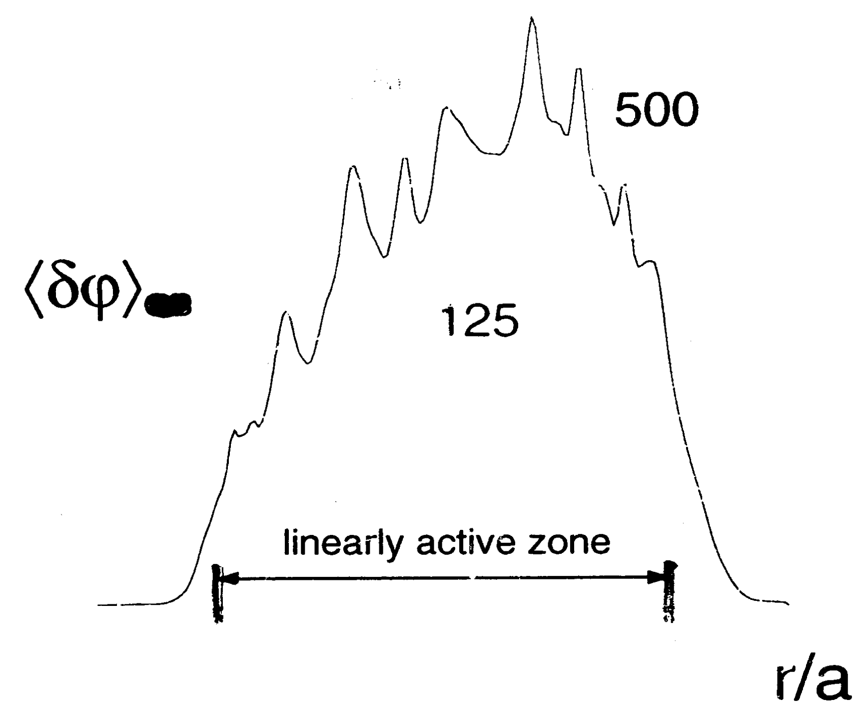
Focus on Weak Turbulence Regime, $\chi_i = \chi_0 I$
as observed in gyrokinetic simulation.



Local excitation rate $\gamma(x)$ as a function of radius.

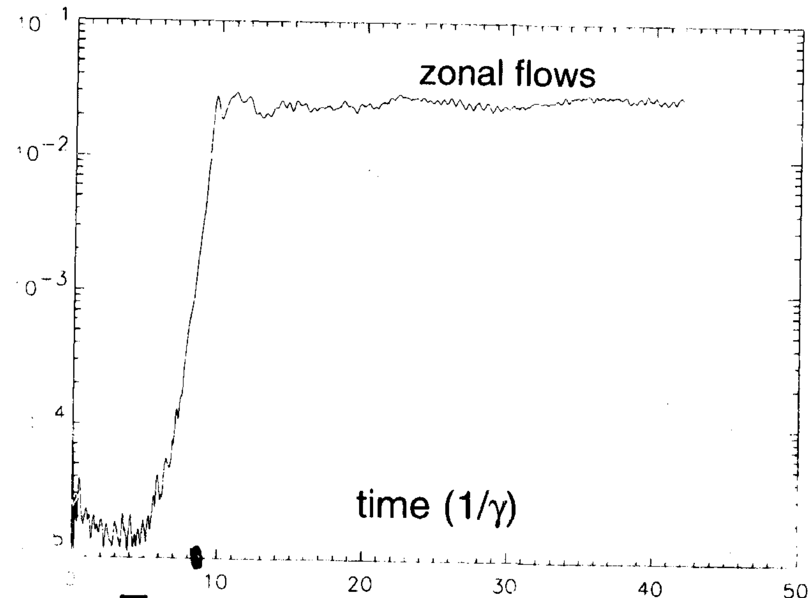
Radial Expansion of Active Turbulence Zone

- In the nonlinearly saturated phase, fluctuations spread radially in both directions.
- Radial expansion $\sim 25\rho_i$ independent of a.



Linear Stability　S ⋮ u ⋮ S

Lin et al., TH 1/1 : Lyon IAEA

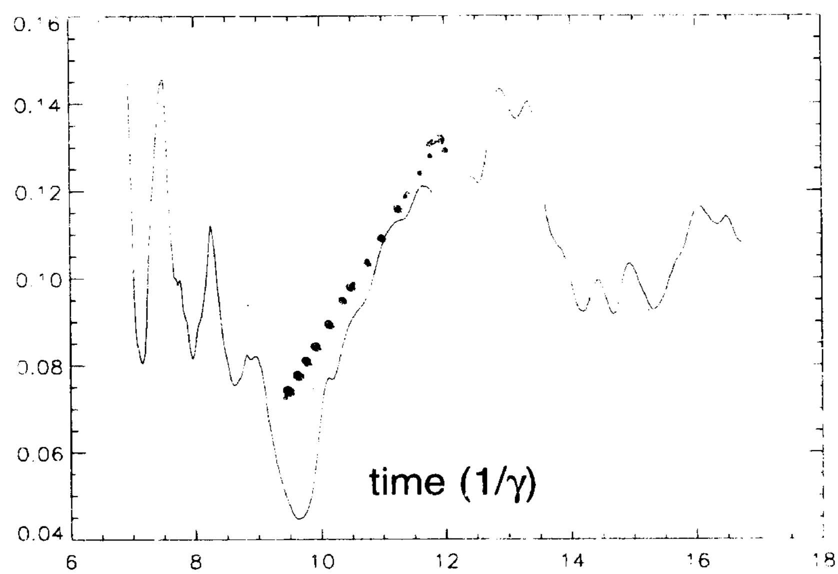


I   II.

ıase :
I. fluct$^{\underline{n}}$ ↑   ZF ↑ → ^{NL}Sat$^{\underline{n}}$

hase :
II. Self-regulated 2 comp. System.
∴ Radial Spreading Occurs in this phase.

- Fluctuation Envelope Radial Width



- Better Diagnostics Needed.
  (Def. of fluct^n prop. front)

from
GK Simulation.

Propagation of Fluctuation Front

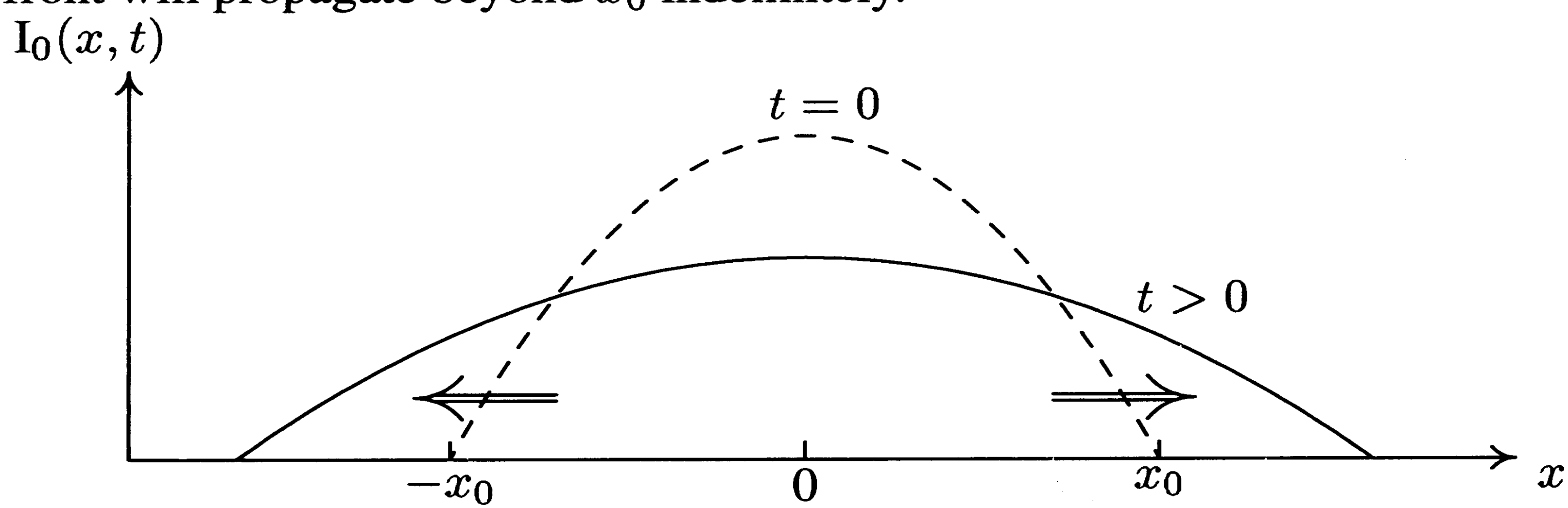
For $\gamma(x) \simeq 0$ and $I << 1$, concentrate on the **nonlinear** diffusion:

$$\frac{\partial}{\partial t} I_0 = \chi_0 \frac{\partial}{\partial x}(I_0 \frac{\partial}{\partial x} I_0)$$

With an initial profile $I_0(x,0) = \frac{\epsilon}{x_0}(1 - \frac{x^2}{x_0^2})H(x_0 - x)$,
An exact solution is well-known (Barenblatt, '79).

$$I_0(x,t) = \frac{\epsilon}{(6\epsilon\chi_0 t + x_0^3)^{1/3}}(1 - \frac{x^2}{(6\epsilon\chi_0 t + x_0^3)^{2/3}})H$$

The front will propagate beyond x_0 indefinitely.



Short Term Behavior: Ballistic Propagation

- $x_{front} = (x_0^3 + 6\epsilon\chi_0 t)^{1/3}$
- $U_x = \frac{d}{dt} x_{front}$
  $\sim 2\epsilon\chi_0 / x_0^2$: for small t (consequence of $\Delta << x_0$)
  $\sim t^{-2/3}$: for large t (sub-diffusion)

  Note: $\epsilon \propto I$, turbulence intensity

- Scaling of U_x drastically different from
  $\mathbf{V}_{gr}$ of linear drift (ITG) wave.

  $\rightarrow$ contrast our theory from others relying on linear dispersion
  [eg., Garbet '94, Chen *et al.*, '03]

Turbulence Spreading has been widely observed

- X. Garbet *et al.*, NF '94 (Mode-coupling in Torus without Zonal Flows)

- From Global Gyrokinetic Particle Simulations:

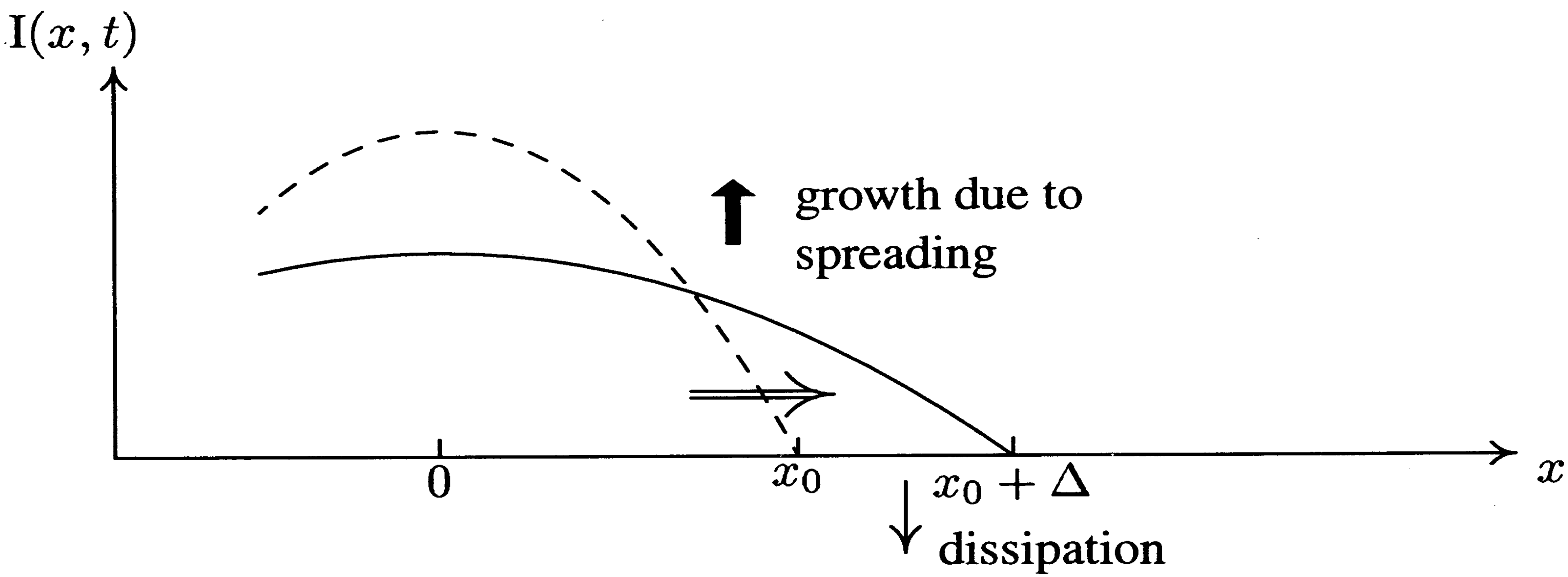
  R. Sydora *et al.*, PPCF '96 (Torus with Zonal Flows)
  Y. Kishimoto *et al.*, PoP '96 (Torus with Zonal Flows)
  S. Parker *et al.*, PoP '96 (Torus without Zonal Flows)
  W.W. Lee *et al.*, PoP '97 (Torus without Zonal Flows)
  Y. Idomura *et al.*, PoP '00 (Sheared Slab with Zonal Flows)
  Z. Lin *et al.*, PRL '02 (Torus with Zonal Flows:**Scaling Studies**)
  L. Villard *et al.*, NF '03
  (Cylinder with Zonal Flows and Profile Evolution)

- Neither Zonal Flows nor Toroidal Eigenmodes necessary
  for Turbulence Spreading.

Saturation of Propagation Front

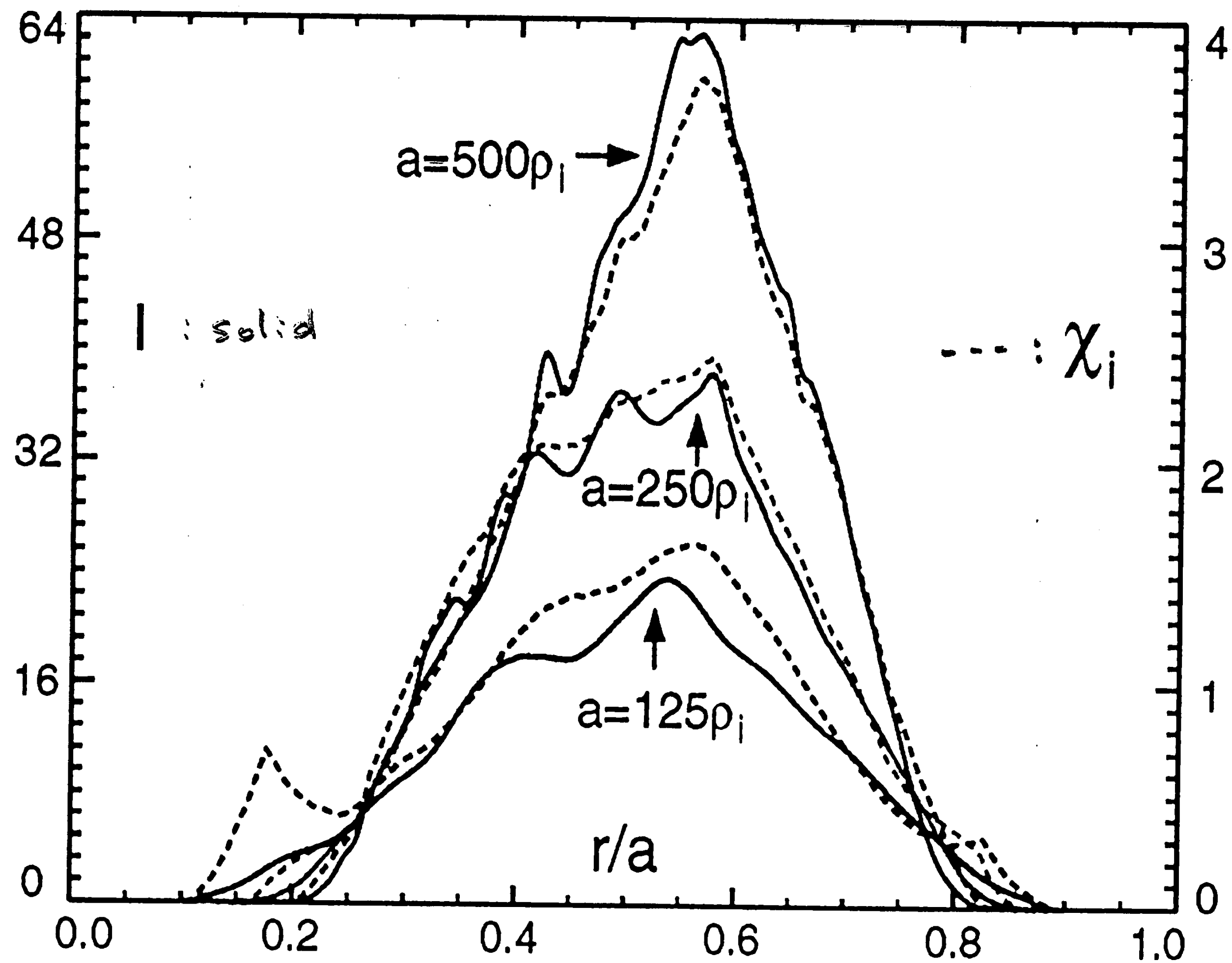
The front propagation stops
when radial flux due to propagation is balanced by dissipation:

$$T_{prop} \simeq \Delta/U_x <<\!\!-\!\!\!->> T_{damp} \sim (|\gamma'|\Delta)^{-1}$$



$\Delta^2 \simeq \frac{12\epsilon\chi_0}{|\gamma'|x_0^2}$, using the values from simulation $\rightarrow \Delta \simeq 18\rho_i$
From GK simulation: $\Delta \simeq 25\rho_i$.

$\Delta/a \searrow$ as $a \nearrow$: $\Delta \simeq 25\rho_i$



Simulation Stimulates Theory Development

PPPL

[Lin et al., PRL '02, Hahm, APS invited '01]

- Deviation from gyroBohm scaling, while $\Delta r \simeq 7\rho_i$

- Radial spreading of turbulence into linearly stable zone

$\rightarrow \chi_i \propto |\delta\phi|^2$, from [Lin et al., PRL '99]

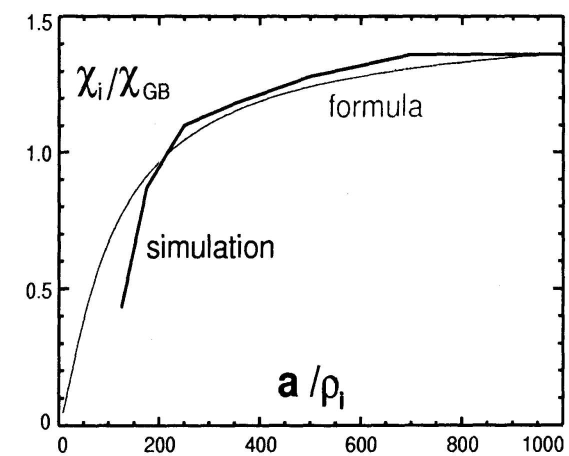
$\rightarrow \chi_{\text{gyroBohm}}$ as $\rho_* \rightarrow 0$

* $\rightarrow$ Additional assumption:

Total fluctuation energy content

not affected by radial spread

$$\chi_i \simeq \chi_{\text{gyroBohm}}/(1 + 100\rho_*)$$

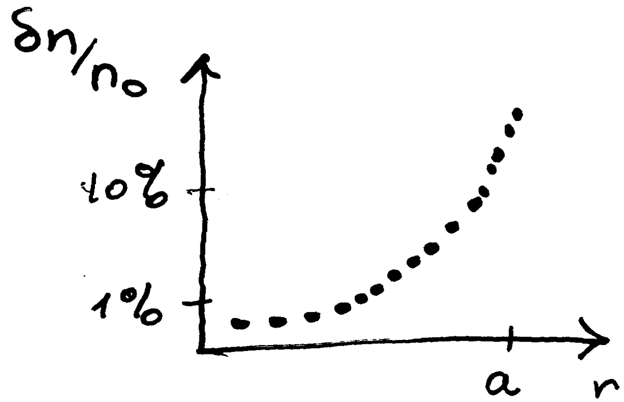


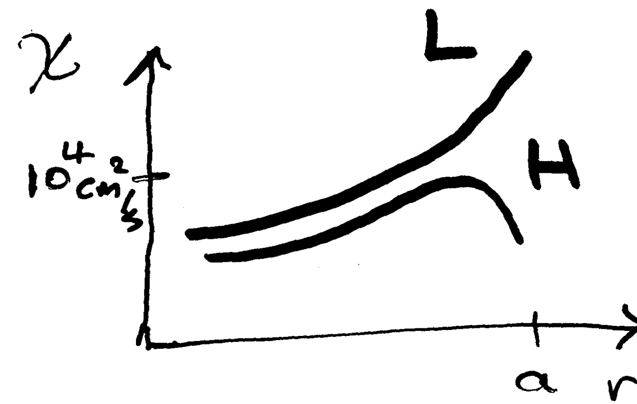
Why Bohm-like ?

* → Dynamical Model : IAEA 2002

ITPA ''

• Turbulence Spreading from Edge to Core :





* Profile of Turb. Intensity crucial in turb. spreading

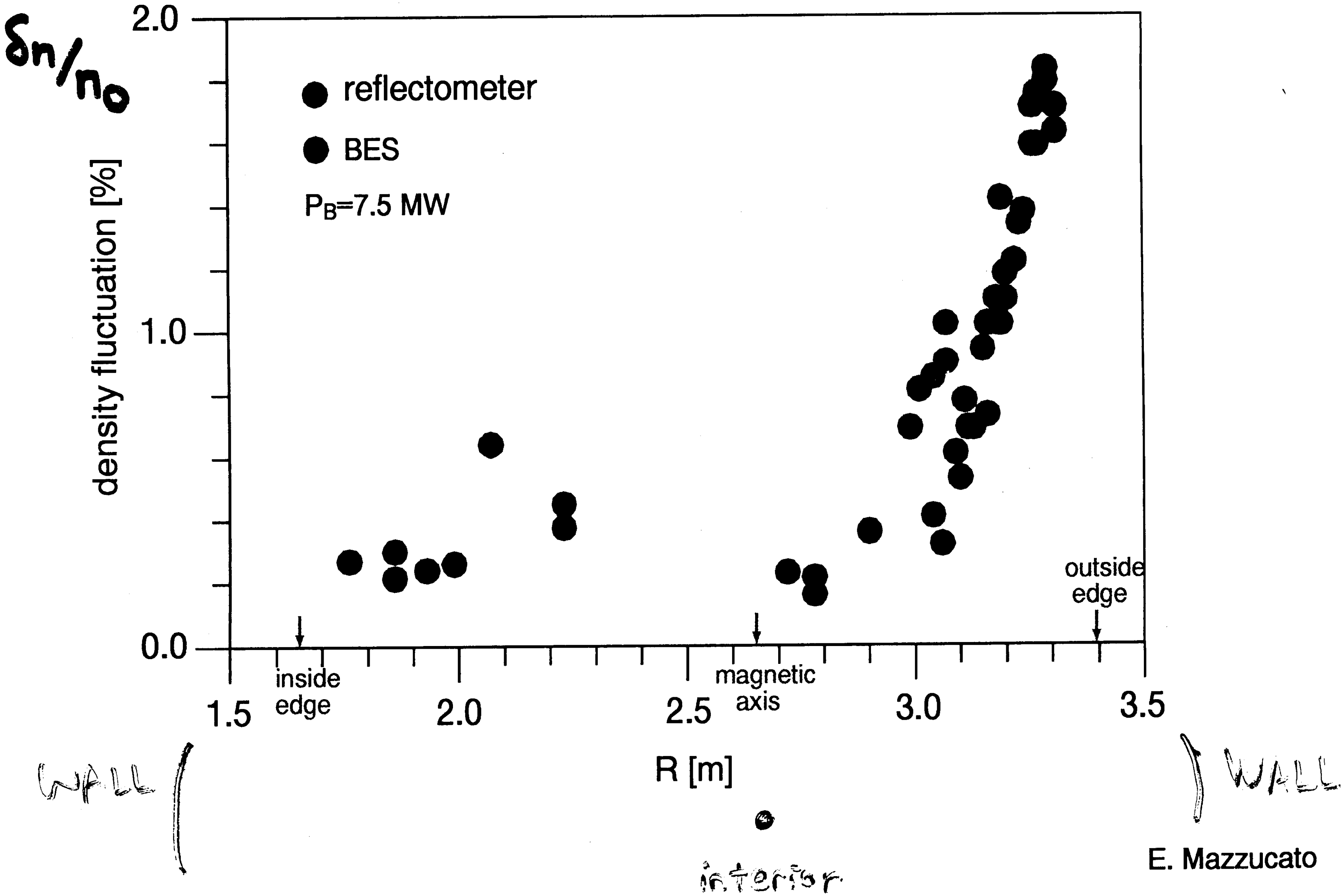
$$\frac{\partial}{\partial t} I = -\frac{\partial}{\partial x}\Gamma_I \; ; \quad \text{"} \Gamma_I = -\chi(I)\frac{\partial}{\partial x} I \text{"}$$

* Core confinement improvement after L-H transition !?

* → Attention to "strongly turb. connection region between Edge & Core":

(cf. V. Parail, PPCF '02)

recent results on ballooning structrure of turbulence in TFTR



E. Mazzucato

* Turb. Spreading could be important in Pedestal Phys.

- Recall: Turb. spreading extent, Δ strongly dep. on $\gamma(x)$ "profile"

$$\frac{\partial}{\partial x}\gamma(x) \sim \frac{\partial^2}{\partial x^2}P$$: "large near pedestal"

- Turb. in connection region:

local drift wave turb. (ITG, etc ...) + Incoming Edge Turbulence

$\Rightarrow$ Desirable to have physics description / capability to cover core - connection region - edge

$\rightarrow$ GK

Tokamak Edge Characteristics

- Large Fluctuation Amplitude in L-mode and OH;
  $\delta n/n_0 \sim e\delta\phi/T_e \sim 10^{-1}$

c.f. $\delta n/n_0 < 10^{-2}$ in TFTR core

- Sharp Gradients in E_r and P in H-mode;
  $\rho_{ip} \sim L_P \sim L_E$

- $\rho_i/L_p \sim 10^{-1} >> L_p/R \sim 10^{-2}$

cf: primary small parameter <—> device-specific

in conventional GK

- Long mean free path $\lambda_{mfp} > 2\pi qR$
  and Large orbits (DIII-D,...) $\rightarrow$ Kinetic Approach

Summary

- Turbulence spreading is widely observed in global gyrokinetic particle simulations.
- **Profile of Fluctuation Intensity** is crucial to its Spatio-temporal Evolution
- Spreading of Edge Turbulence into Core is being studied.
- Coupling between **Core Turbulence and Edge Turbulence** expected to be strong near Pedestal.
- Kinetic approach desirable for future tokamak edge turbulence.
- Nonlinear gyrokinetic formulation for tokamak edge turbulence is near completion.