Full-Wave Maxwell Simulations for ECRH

H. Hojo

Plasma Research Center, University of Tsukuba

in collaboration with

A. Fukuchi, N. Uchida, A. Shimamura, T. Saito and Y. Tatematsu

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Old version of Maxwell Equation Simulator (Maxim)

This code is used mainly for plasma diagnostic problems.

$$\frac{\partial}{\partial t}\boldsymbol{B} = -\nabla\times\boldsymbol{E}$$

$$\frac{\partial}{\partial t}\boldsymbol{E} = \frac{c^2}{\underline{\varepsilon(\boldsymbol{r})}}\nabla\times\boldsymbol{B} - \frac{1}{\varepsilon_0\varepsilon(\boldsymbol{r})}[\boldsymbol{J} + \underline{\sigma(\boldsymbol{r})\boldsymbol{E}}]$$

Artificial terms introduced to set wall boundary

$$\frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\boldsymbol{J} = \omega_{\rm pe}^2[n(\boldsymbol{r})]\boldsymbol{E} - \frac{e}{\varepsilon_0 m_{\rm e}}\boldsymbol{J}\times\boldsymbol{B}_0(\boldsymbol{r})$$

Input:

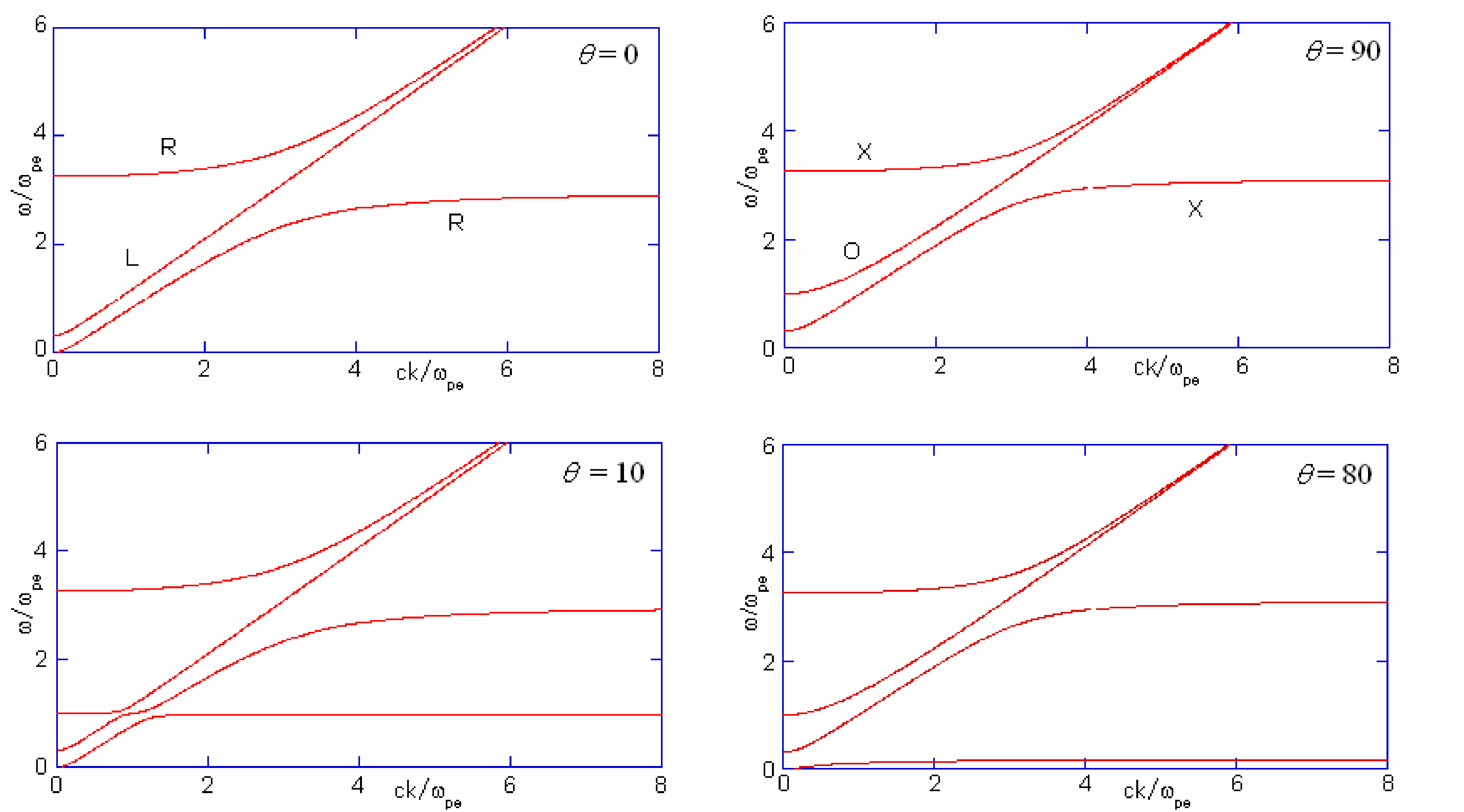
$n(\boldsymbol{r})$ = **Plasma density (arbitrary)**

$\boldsymbol{B}_0(\boldsymbol{r})$ = **External magnetic field (arbitrary)**

$$\sigma(\boldsymbol{r}) = \begin{cases} 0 \\ \sigma_*(>0) \end{cases} \quad \varepsilon(\boldsymbol{r}) = \begin{cases} 1 & \textbf{plasma} \\ \varepsilon_*(>1) & \textbf{otherwise} \end{cases}$$

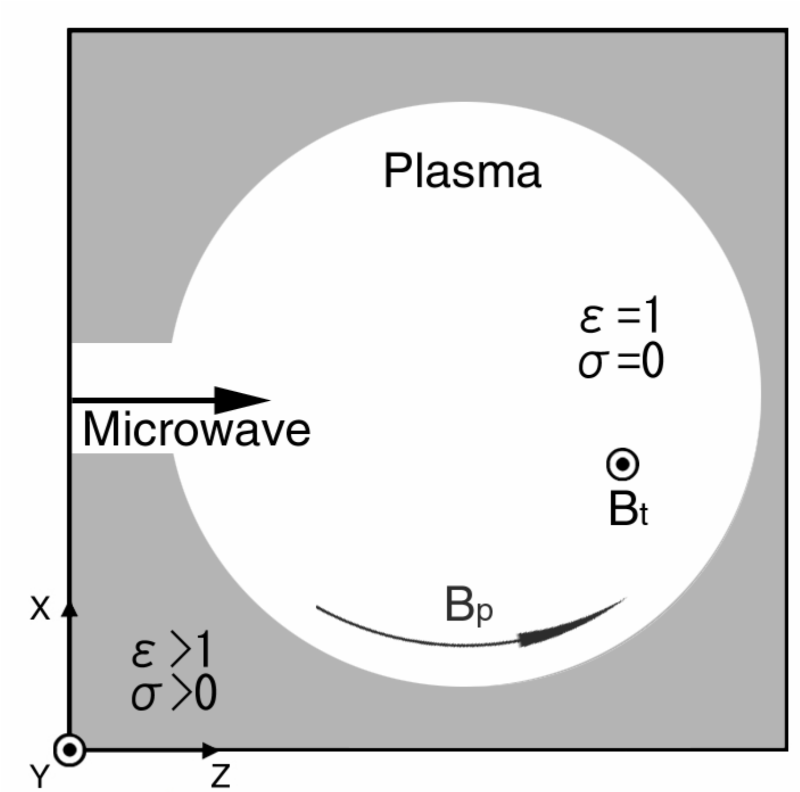
Incident wave : arbitrary (cw, pulse, etc)

A set of equations($m_\mathrm{i} = \infty$) for simulation can describe
R- and L-modes in parallel propagation, and
O and X-modes in perpendicular propagation
in high frequency range (GHz or more).

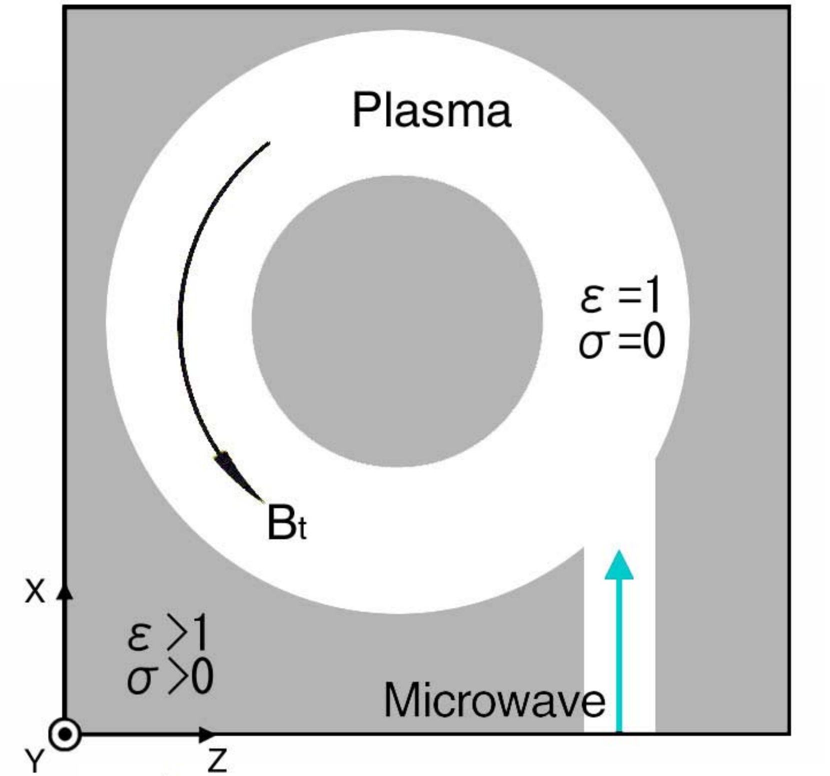


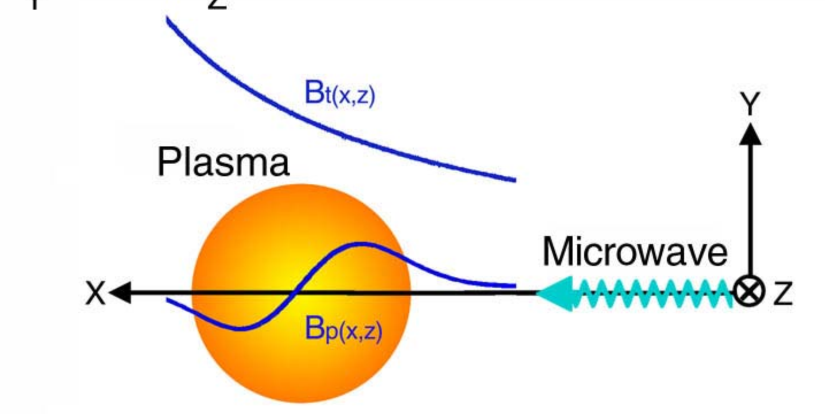
Two 2-d simulation models on EM wave propagation

Sectional View

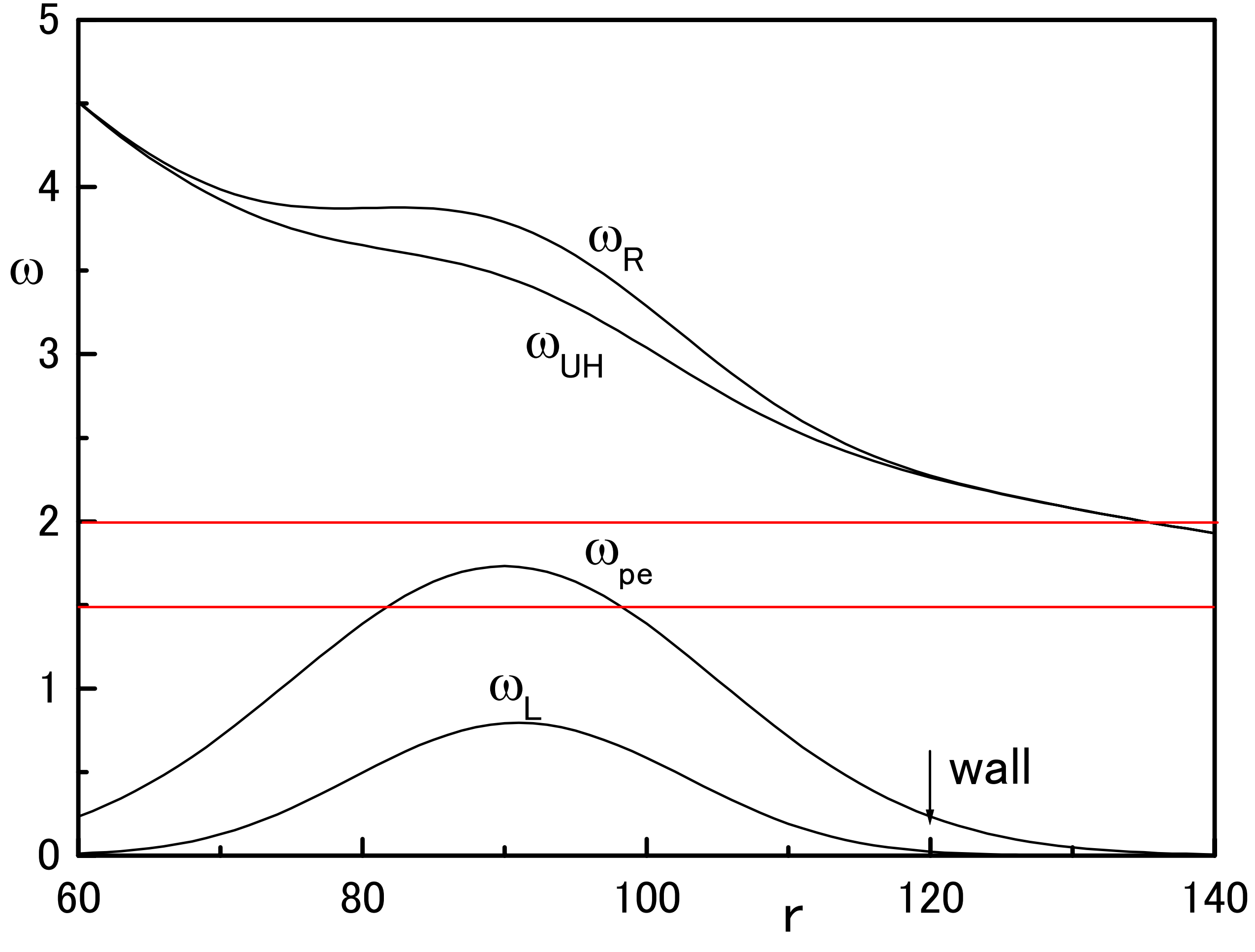


Top View





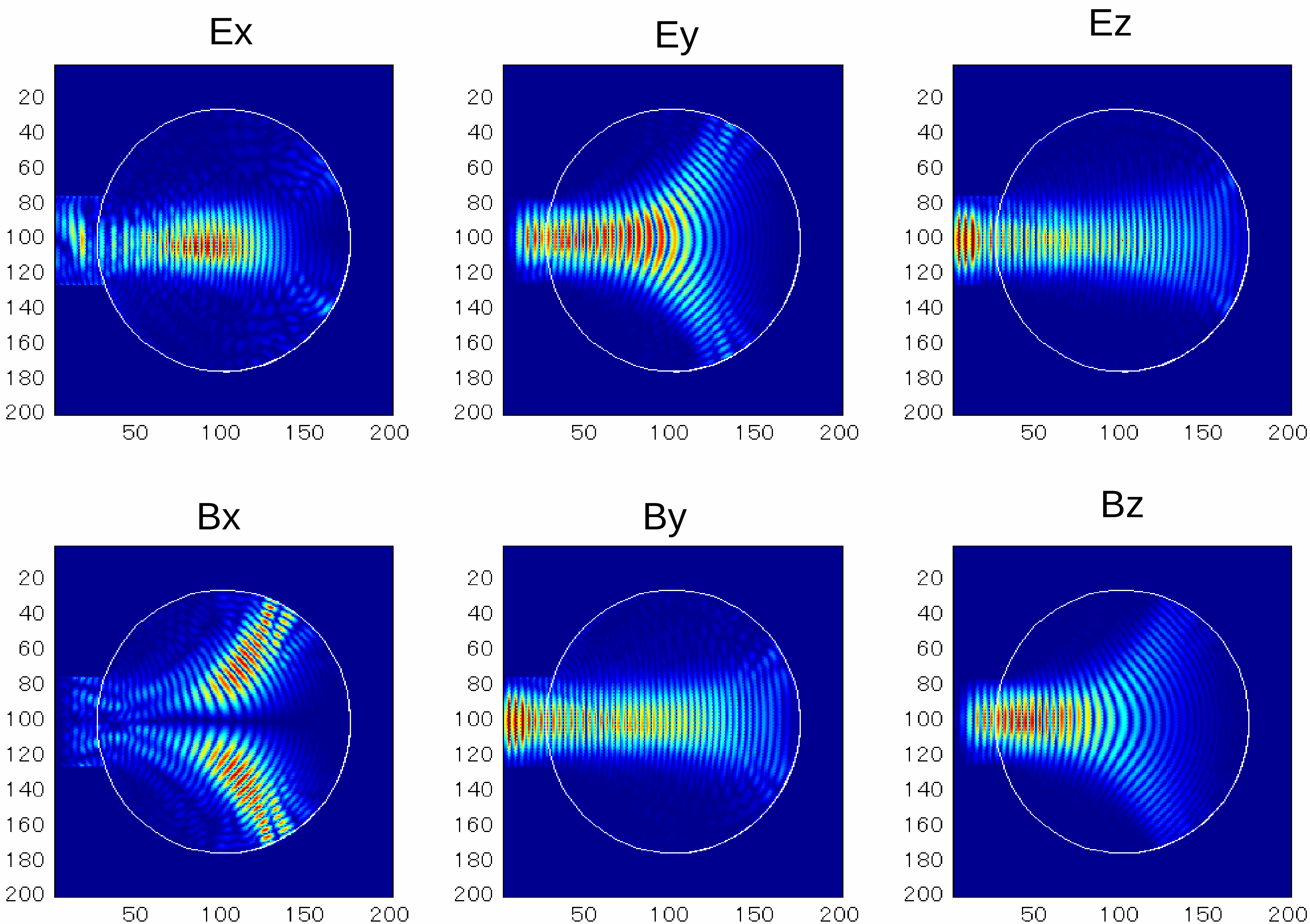
Radial Profiles of Characteristic Frequencies in Tokamak Model



$f_0=3$
$\alpha=3$

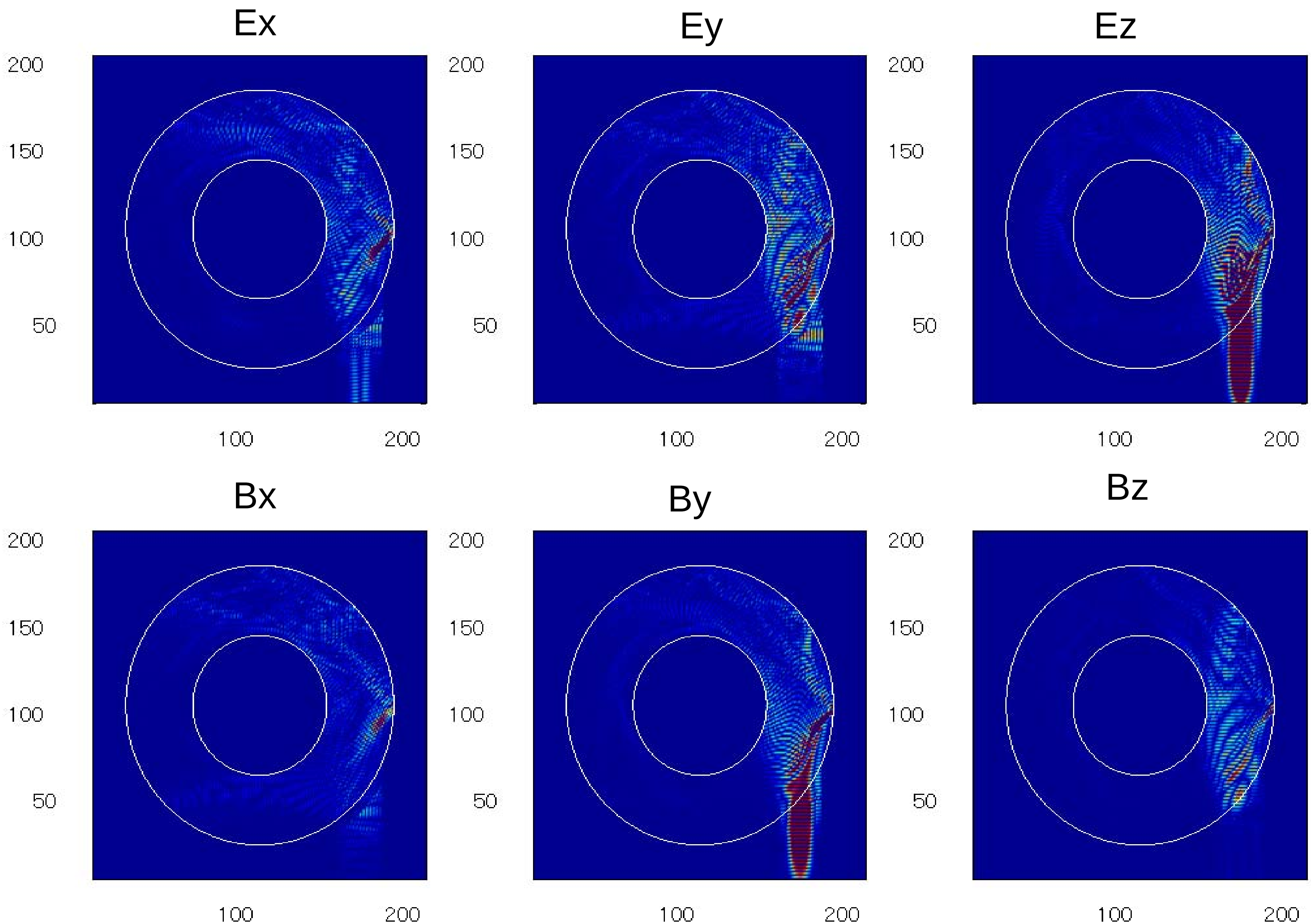
EM Wave Propagation (Vertical Injection)

at $\omega/\omega_0 = 2$ and $(\omega_{pe0}/\omega_0)^2 = 3$

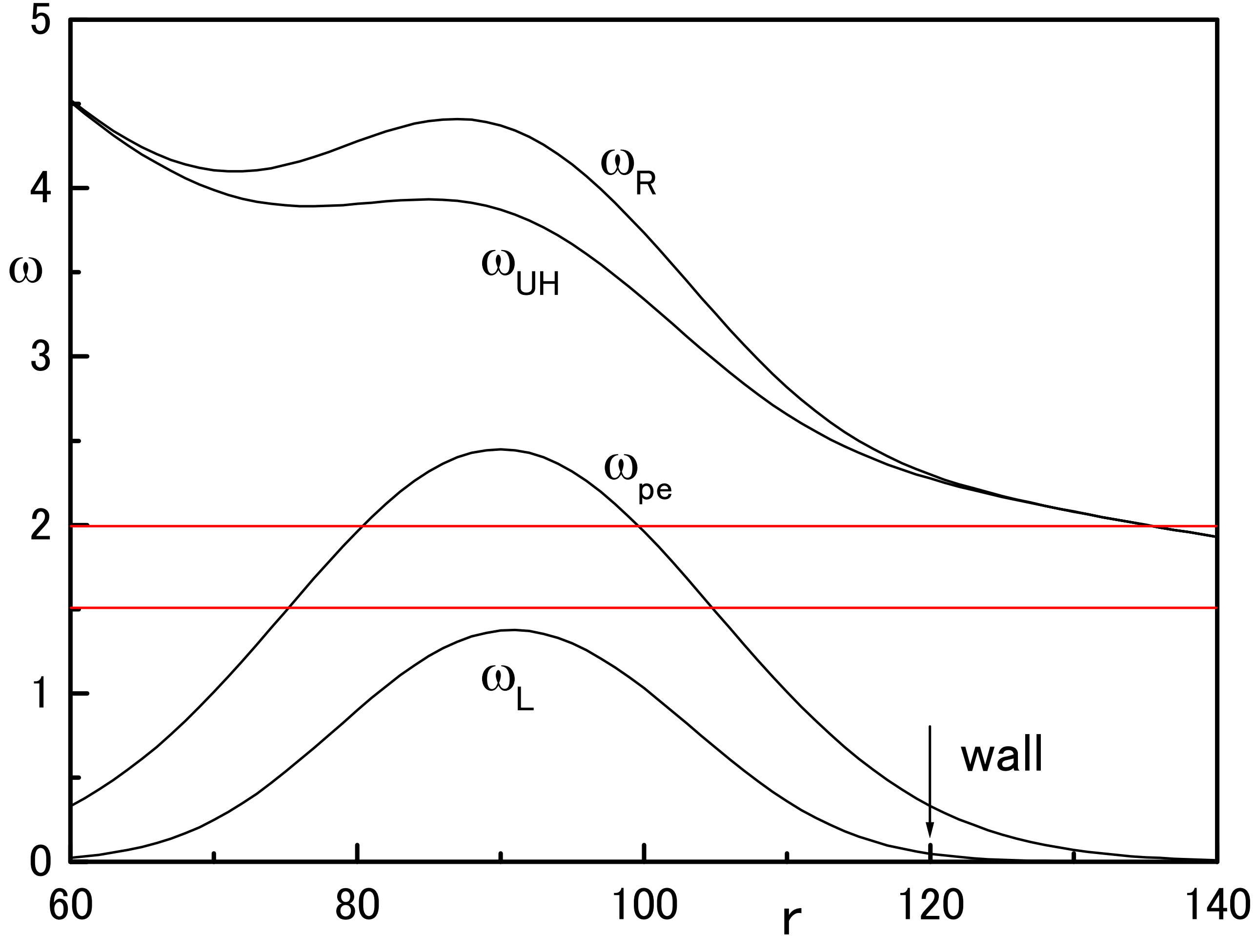


EM Wave Propagation (Tangential Injection)

at $\omega/\omega_0 = 2$ and $(\omega_{pe0}/\omega_0)^2 = 3$



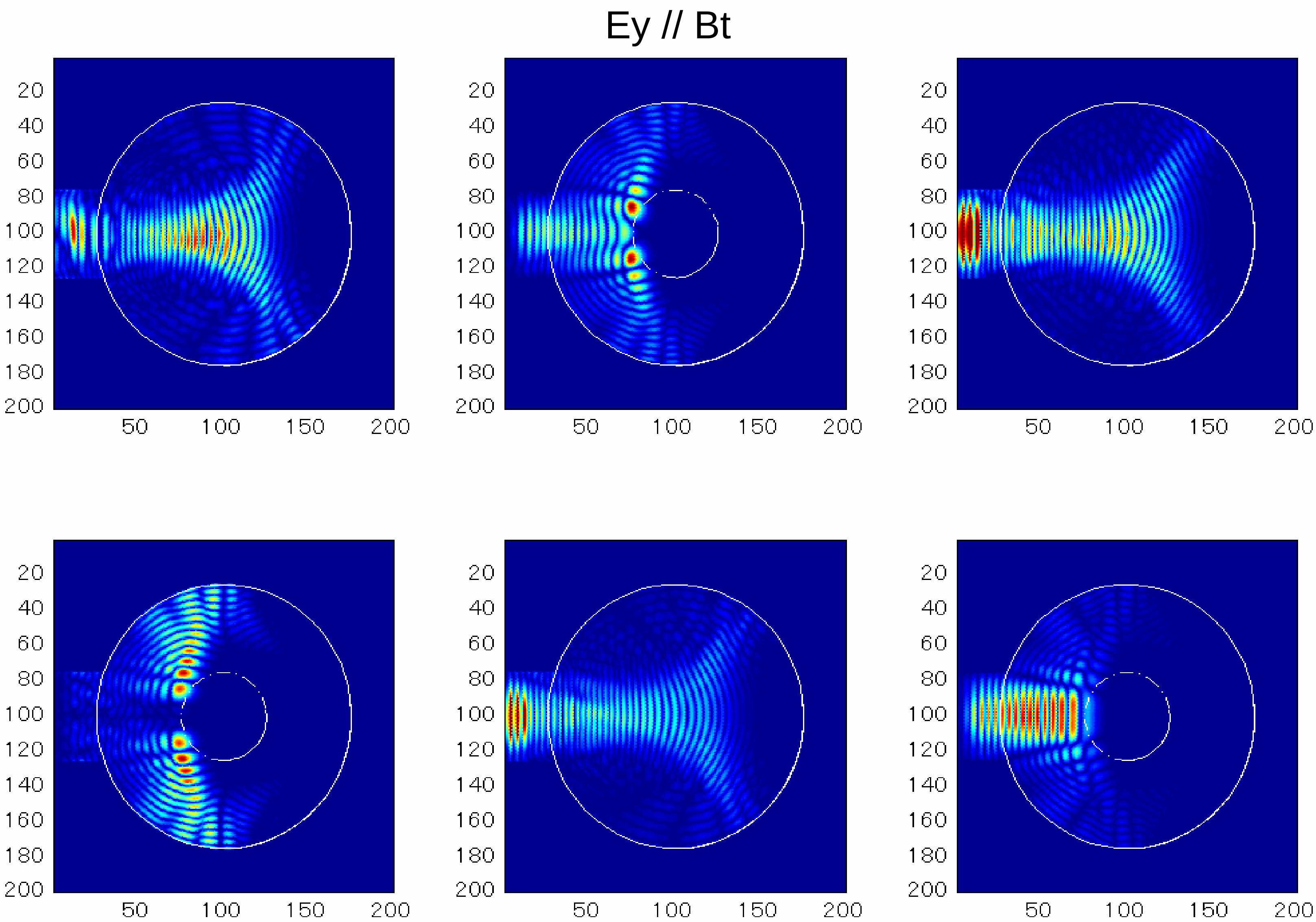
Radial Profiles of Characteristic Frequencies in Tokamak Model



$f_0=6$
$\alpha=3$

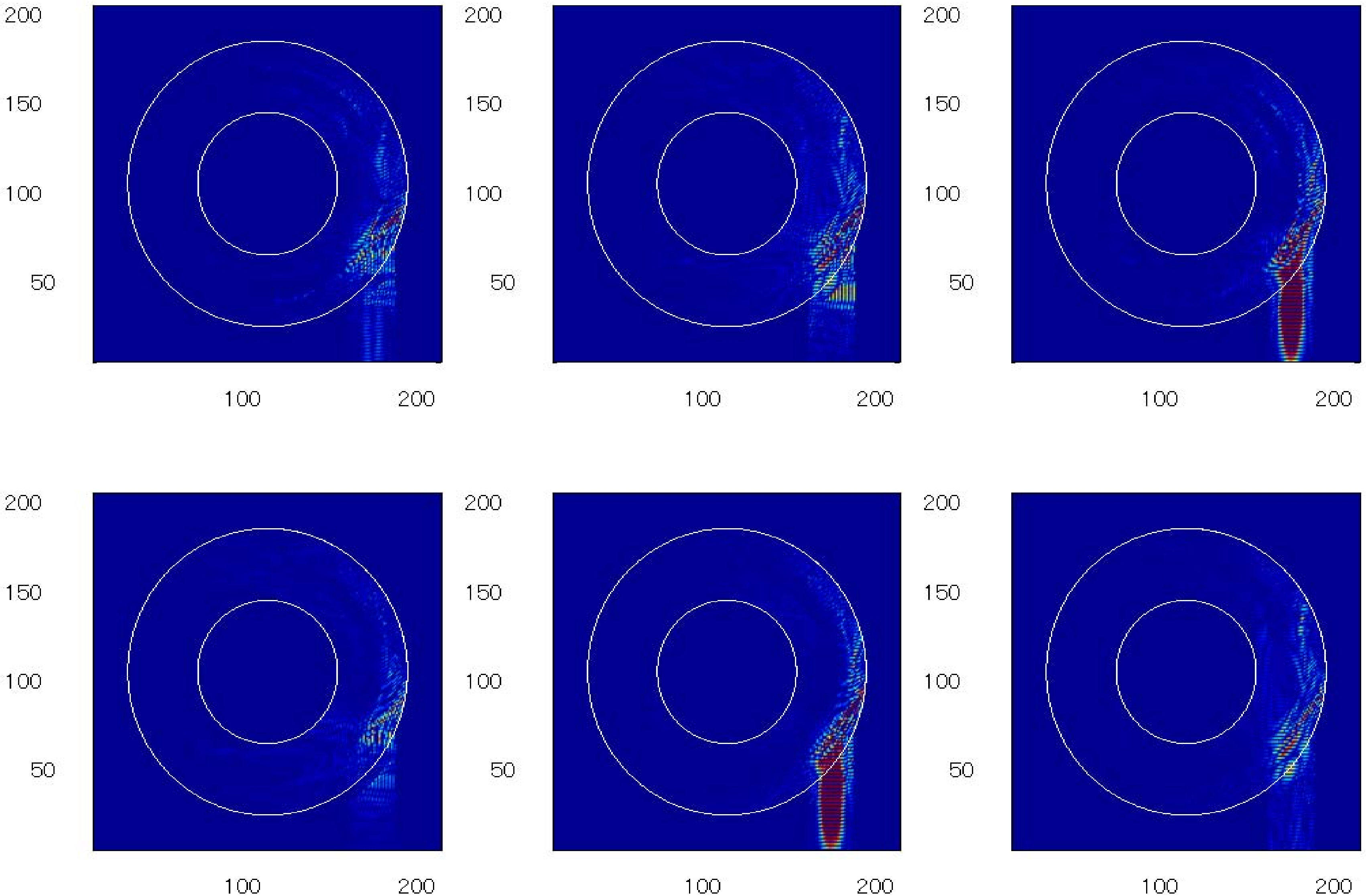
EM Wave Propagation (Vertical Injection)

at $\omega/\omega_0 = 2$ and $(\omega_{pe0}/\omega_0)^2 = 6$



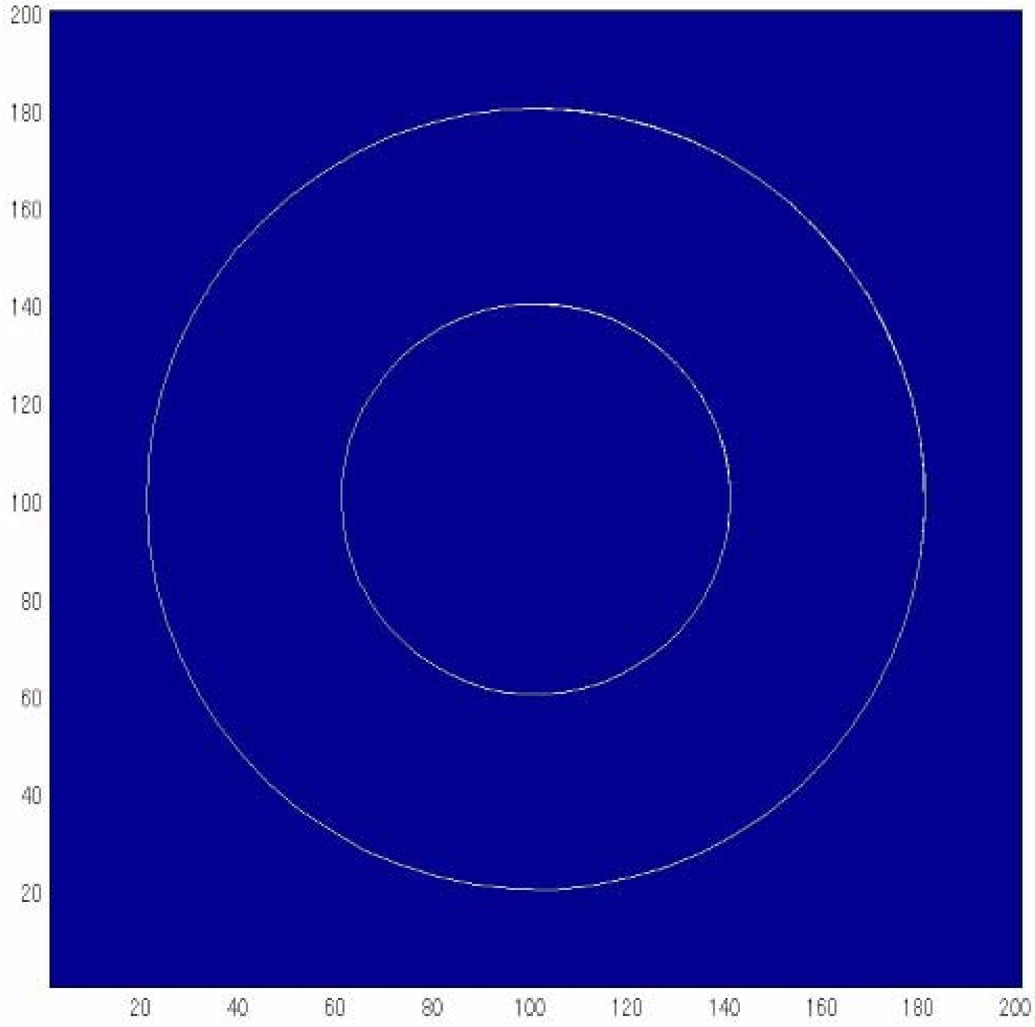
EM Wave Propagation (Tangential Injection)

at $\omega/\omega_0 = 2$ and $(\omega_{pe0}/\omega_0)^2 = 6$

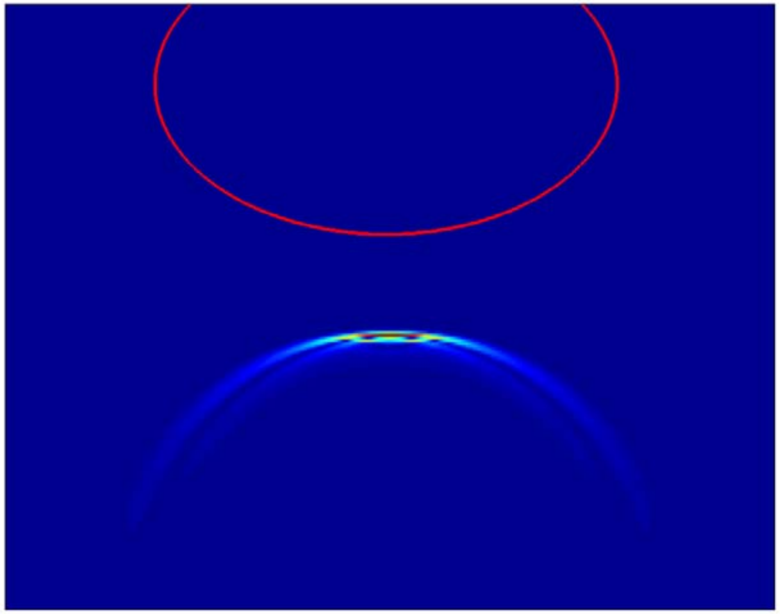


Toroidal propagation of electromagnetic wave

(Movie)



Ultrashort-pulse propagation in LHD plasma



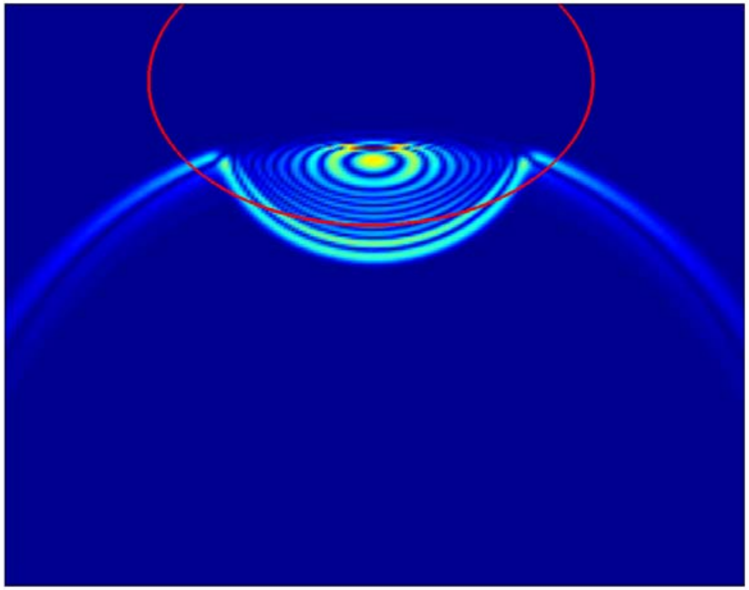
EM Pulse of FWHM = 30ps

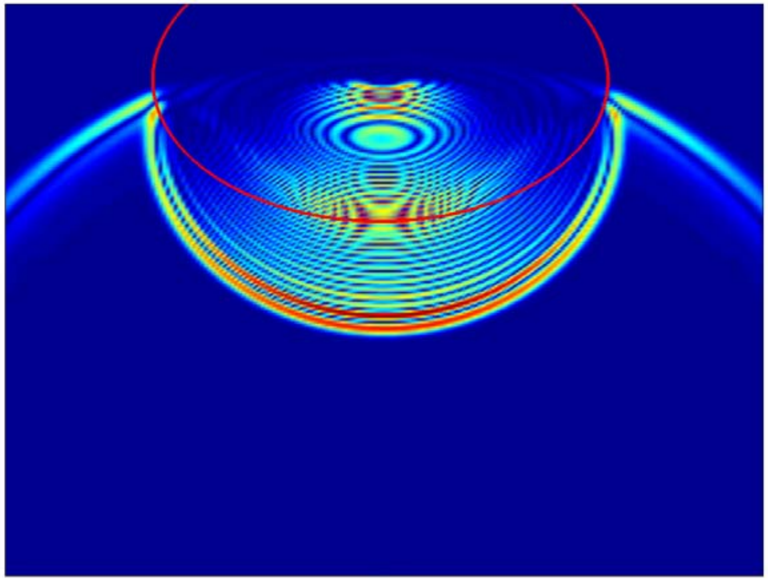
$$n(X,Y) = n_0[1-\psi(X,Y)/\psi_0], \quad n_0 = 5\times10^{12}\,\mathrm{cm}^{-3}$$

$\boldsymbol{B}(X,Y,\phi=0)$ with p=5

[T. Watanabe and H. Akao: J. Plasma Fusion Res. **73** (1997) 186.]

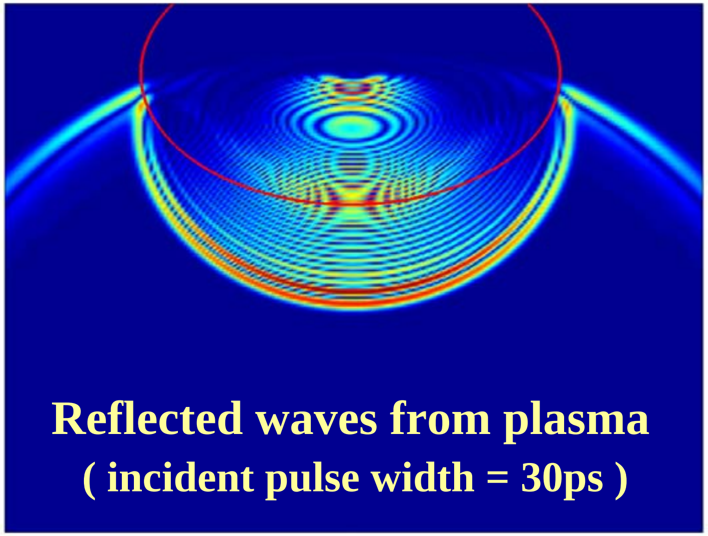
Red ellipse denotes plasma-vacuum boundary.

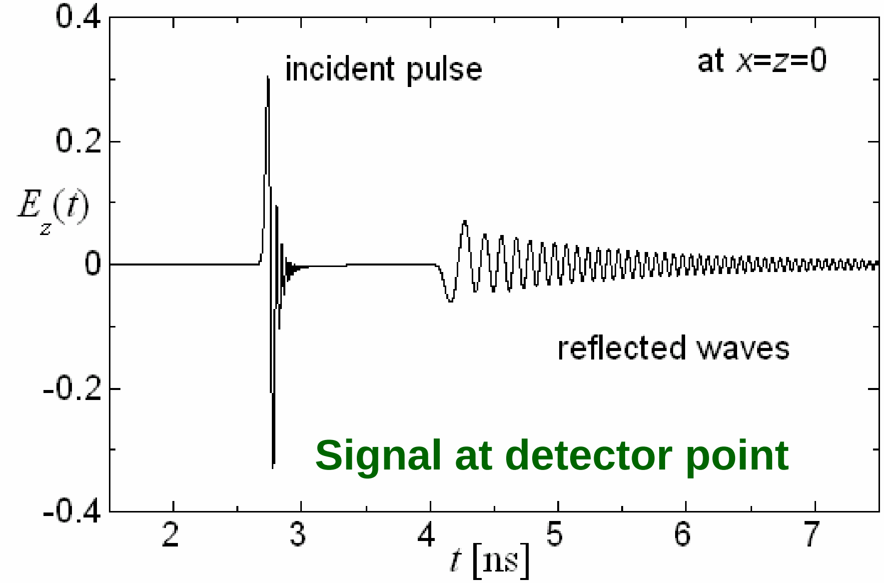


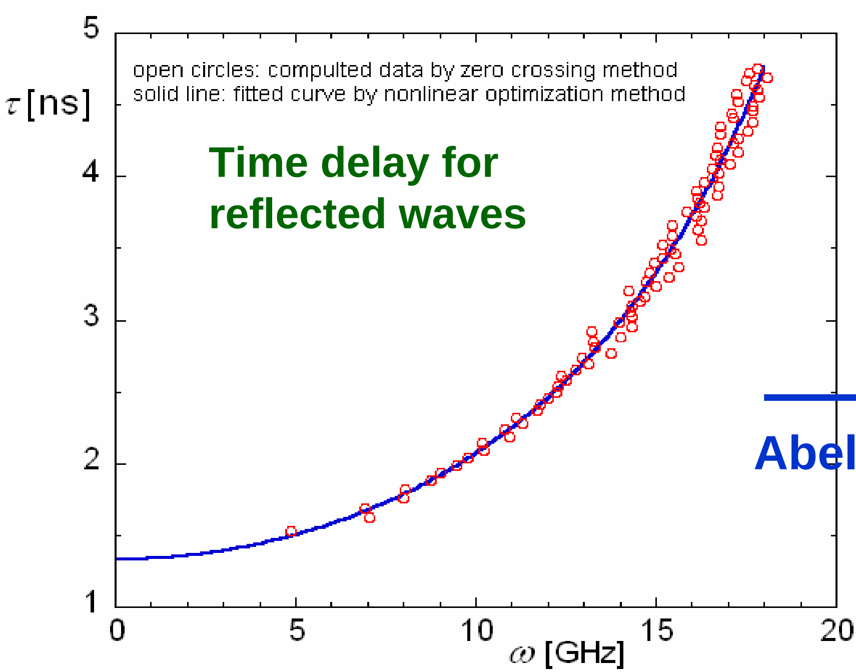


Ultrashort-Pulse Reflectometry for LHD Plasma

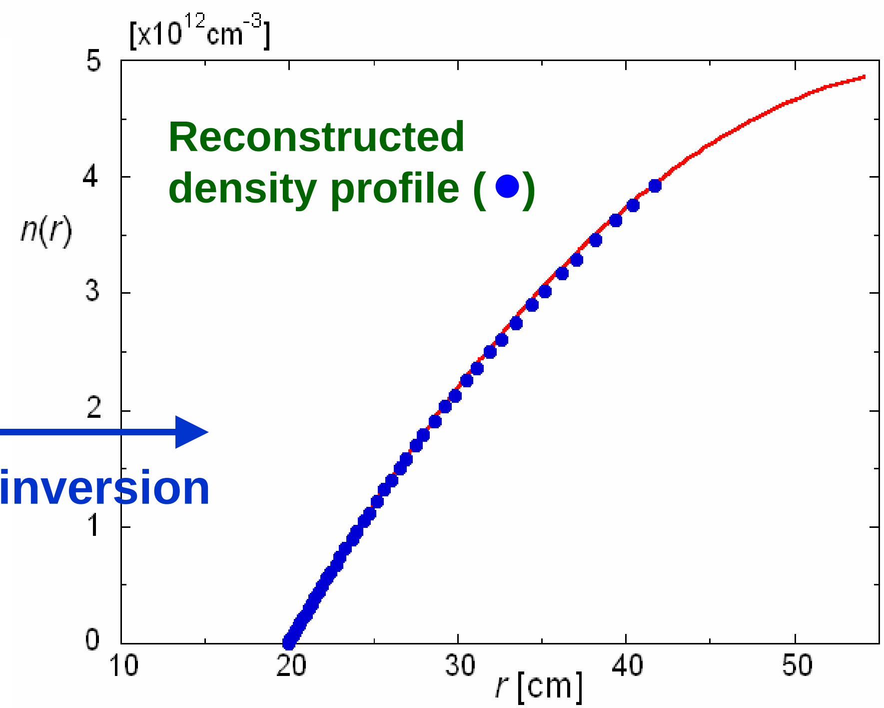
(FDTD Simulation)







Abel inversion



Objective

Development of a new type of computational tool for wave heating in place of ray-tracing method

Ray-tracing method based on geometrical optics is simple, but cannot treat
1. wave diffraction,
2. wave tunneling,
3. mode conversion.

Maxwell equation simulator based on electromagnetic optics can clear the above difficulty in ray-tracing method.

Ray Tracing Method

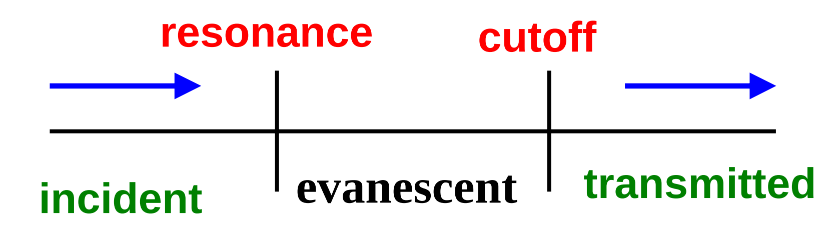
Dispersion Relation: $D(\omega, \boldsymbol{k}, \boldsymbol{r}) = 0$

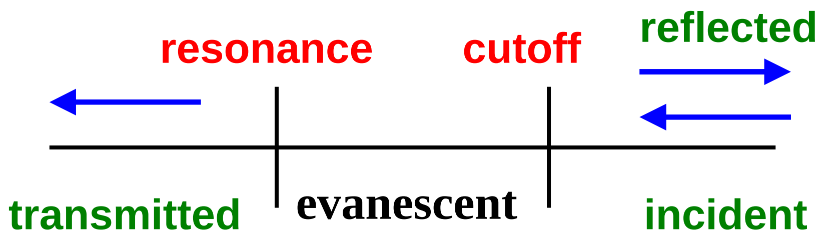
$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{r} = -\frac{\partial D / \partial \boldsymbol{k}}{\partial D / \partial \omega} \,, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{k} = \frac{\partial D / \partial \boldsymbol{r}}{\partial D / \partial \omega}$$

$$W = \frac{1}{2}\mathrm{Re}\left\{ \boldsymbol{E}^* \cdot \overleftrightarrow{\sigma} \cdot \boldsymbol{E} \right\}$$

$$\left|\boldsymbol{E}\right|^2 = \left|\boldsymbol{E}_0\right|^2 \exp(-2\int \mathrm{Im}(k)\mathrm{d}s)$$

Maxwell Equation Simulator





Good Points:

1. Full wave analysis
2. Wave diffraction
3. Wave tunneling
4. Mode conversion

Bad Points:

1. Applicable to fundamental heating

   (Harmonic resonances can be included in the simulator equations, however, which cannot be solved in time domain as initial value problems.)

   To trace waves, it is of importance that wave equations are solved in time domain as initial value problems.

2. Approximated kinetic effect

   (Ray tracing results could be covered.)

Maxwell Wave Equation:

$$\nabla\times\nabla\times\boldsymbol{E}-\left(\frac{\omega}{c}\right)^2\left[1+\mathrm{i}\frac{\overleftrightarrow{\sigma}}{\varepsilon_0\omega}\right]\boldsymbol{E}=0$$

$$\boldsymbol{J}=\overleftrightarrow{\sigma}\cdot\boldsymbol{E}$$

Conductivity tensor $\overleftrightarrow{\sigma}$ is generally obtained from Vlasov equation.

In the limit of $k_\perp\to 0\ \ (\omega\approx|\omega_{\mathrm{ce}}|)$:

$$\mathrm{i}\frac{\overleftrightarrow{\sigma}}{\varepsilon_0\omega}=\begin{pmatrix} S-1 & -\mathrm{i}D & 0\\ \mathrm{i}D & S-1 & 0\\ 0 & 0 & P-1\end{pmatrix}$$

$$S=1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2-\omega_{\mathrm{ce}}^2}+\mathrm{i}\sqrt{\pi}\frac{\omega_{\mathrm{pe}}^2}{2\omega|k_{||}|v_{\mathrm{te}}}\exp\left[-\left(\frac{\omega-|\omega_{\mathrm{ce}}|}{|k_{||}|v_{\mathrm{te}}}\right)^2\right]$$

$$D=\frac{\omega_{\mathrm{ce}}}{\omega}\frac{\omega_{\mathrm{pe}}^2}{\omega^2-\omega_{\mathrm{ce}}^2}+\mathrm{i}\sqrt{\pi}\frac{\omega_{\mathrm{pe}}^2}{2\omega|k_{||}|v_{\mathrm{te}}}\exp\left[-\left(\frac{\omega-|\omega_{\mathrm{ce}}|}{|k_{||}|v_{\mathrm{te}}}\right)^2\right]$$

$$P=1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2}+\mathrm{i}\sqrt{\pi}\frac{\omega\omega_{\mathrm{pe}}^2}{(|k_{||}|v_{\mathrm{te}})^3}\exp\left[-\left(\frac{\omega}{|k_{||}|v_{\mathrm{te}}}\right)^2\right]$$

Approximation : $k_{||}$ in S, D and P is determined by local dispersion equation.

$$D(k_{||},\boldsymbol{r},\omega)=0$$

Equations for Simulations:

Previous conductivity tensor is covered by the following equations.

(These time-evolution equations can be solved as initial value problems.)

$$\frac{\partial}{\partial t}\boldsymbol{B} = -\nabla \times \boldsymbol{E}$$

$$\frac{\partial}{\partial t}\boldsymbol{E} = c^2 \nabla \times \boldsymbol{B} - \frac{1}{\varepsilon_0}\boldsymbol{J} - \frac{1}{\varepsilon_0}\underline{\overset{\leftrightarrow}{\sigma}\cdot \boldsymbol{E}}$$

Cyclotron and Landau damping

$$\frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\boldsymbol{J} = \omega_{\mathrm{pe}}^2 \boldsymbol{E} - \frac{e}{\varepsilon_0 m_{\mathrm{e}}}\boldsymbol{J}\times \boldsymbol{B}_0 - \underline{\frac{\nu}{\varepsilon_0}}\boldsymbol{J}$$

Collisional damping

$$\overset{\leftrightarrow}{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} = \begin{pmatrix} \sigma_1 b_z^2 + \sigma_3 b_x^2 & -\mathrm{i}\sigma_1 b_z & -(\sigma_1 - \sigma_3) b_x b_z \\ \mathrm{i}\sigma_1 b_z & \sigma_1 & -\mathrm{i}\sigma_1 b_x \\ -(\sigma_1 - \sigma_3) b_x b_z & \mathrm{i}\sigma_1 b_x & \sigma_1 b_x^2 + \sigma_3 b_z^2 \end{pmatrix}$$

$$\frac{\sigma_1}{\varepsilon_0 \omega} = \sqrt{\pi}\,\frac{\omega_{\mathrm{pe}}^2}{2\omega |k_{\|}| v_{\mathrm{te}}}\exp\left[-\left(\frac{\omega - |\omega_{\mathrm{ce}}|}{|k_{\|}| v_{\mathrm{te}}}\right)^2\right]$$

$$\boldsymbol{b} = \frac{\boldsymbol{B}_0}{B_0} = b_x \hat{\boldsymbol{x}} + b_z \hat{\boldsymbol{z}}$$

$$\frac{\sigma_3}{\varepsilon_0 \omega} = \sqrt{\pi}\,\frac{\omega \omega_{\mathrm{pe}}^2}{(|k_{\|}| v_{\mathrm{te}})^3}\exp\left[-\left(\frac{\omega}{|k_{\|}| v_{\mathrm{te}}}\right)^2\right]$$

Absorbed power deposition in ECRH :

$$W = \frac{1}{2}\mathrm{Re}\left\{ \boldsymbol{E}^{*} \cdot \overleftrightarrow{\sigma} \cdot \boldsymbol{E} \right\}$$

$$= \frac{1}{2}\sigma_1 (b_z^2 \left|E_x\right|^2 + \left|E_y\right|^2 + b_x^2 \left|E_z\right|^2) + \frac{1}{2}\sigma_3 (b_x^2 \left|E_x\right|^2 + b_z^2 \left|E_z\right|^2)$$

$$+ \frac{1}{2}(\sigma_5 - \sigma_1) b_x b_z (E_x^* E_z + E_x E_z^*) + \frac{\mathrm{i}}{2}\sigma_1 \left[(b_z E_x - b_x E_z) E_y^* - (b_z E_x^* - b_x E_z^*) E_y \right]$$

When $b_z = 1,\ b_x = 0$

$$W = \frac{1}{2}\sigma_3 \left|E_z\right|^2 + \frac{1}{2}\sigma_1 \left|E_x - \mathrm{i}E_y\right|^2$$

$$= \frac{1}{2}\sigma_3 \left|E_z\right|^2 + \frac{1}{2}\sigma_1 \left(\left|E_x\right|^2 + \left|E_y\right|^2 + 2\,\mathrm{Re}(E_x)\,\mathrm{Im}(E_y) - 2\,\mathrm{Im}(E_x)\,\mathrm{Re}(E_y) \right)$$

Cross Polarization Scattering (O and X-modes)

Time domain version of Fidone-Granata equation: mode conversion by magnetic shear

$$\left(\frac{\partial^2}{\partial t^2}+\omega_{pe}^2+\omega_{ce}^2\right)\frac{\partial}{\partial t}E_x+\omega_{pe}^2\omega_{ce}E_\perp=0$$

$$\left(\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial x^2}+\omega_{pe}^2+c^2\theta'^2\right)E_\perp+\omega_{ce}E_x=c^2\left[\underline{2\theta'}\frac{\partial E_\parallel}{\partial x}+\underline{\theta''}E_\parallel\right]$$

X-mode

$$\left(\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial x^2}+\omega_{pe}^2+c^2\theta'^2\right)E_\parallel=-c^2\left[\underline{2\theta'}\frac{\partial E_\perp}{\partial x}+\underline{\theta''}E_\perp\right]$$

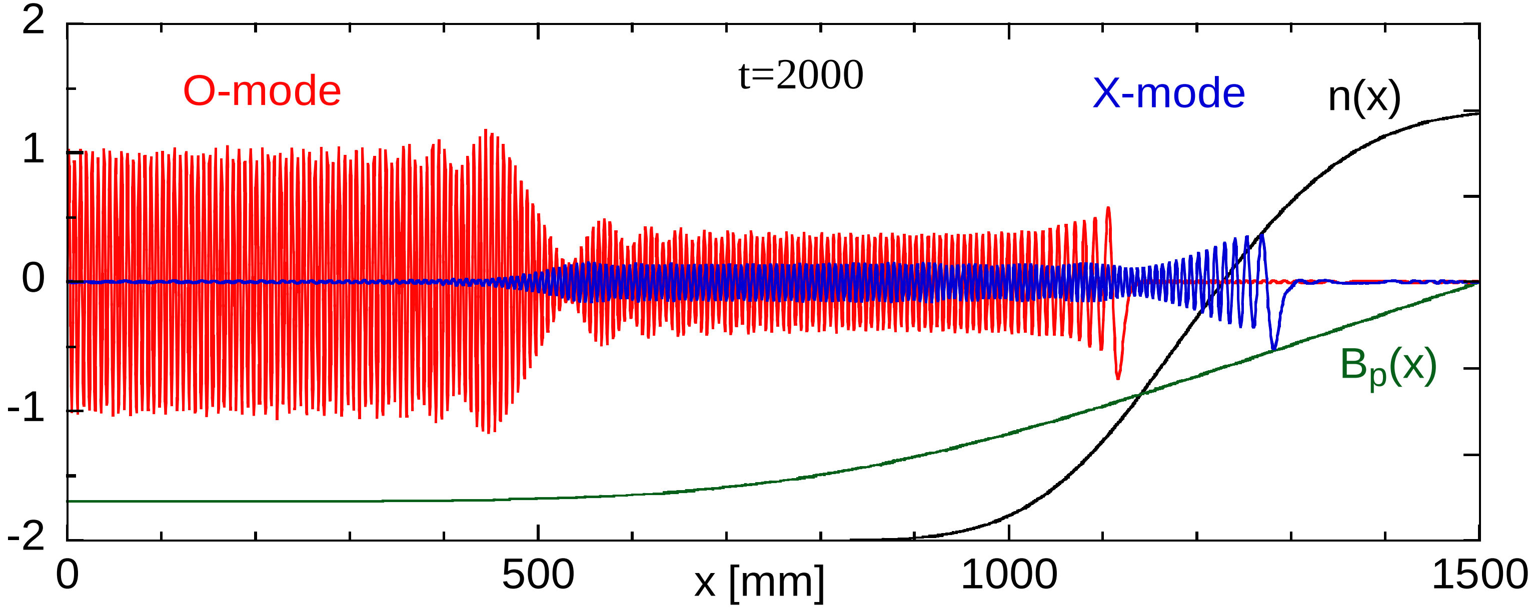
O-mode

$$\boldsymbol{B}(x)=B_z(x)\hat{\boldsymbol{z}}+B_y(x)\hat{\boldsymbol{y}}$$

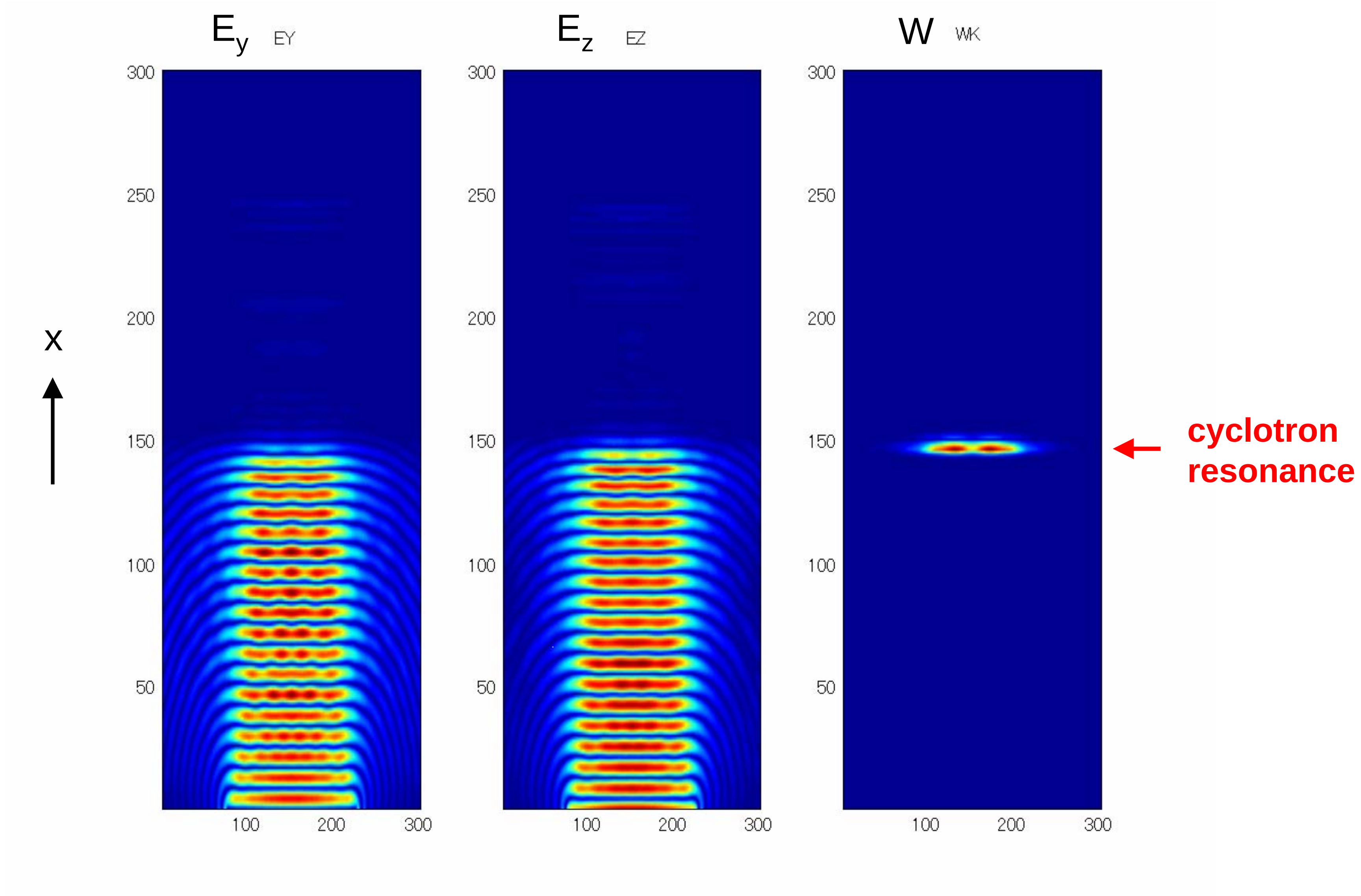
$$\theta(x)=\arctan(B_y/B_z)$$

$$\theta'=\mathrm{d}\theta/\mathrm{d}x,\quad \theta''=\mathrm{d}^2\theta/\mathrm{d}x^2$$

Incident O-mode to X-mode at 48GHz in LHD Plasma



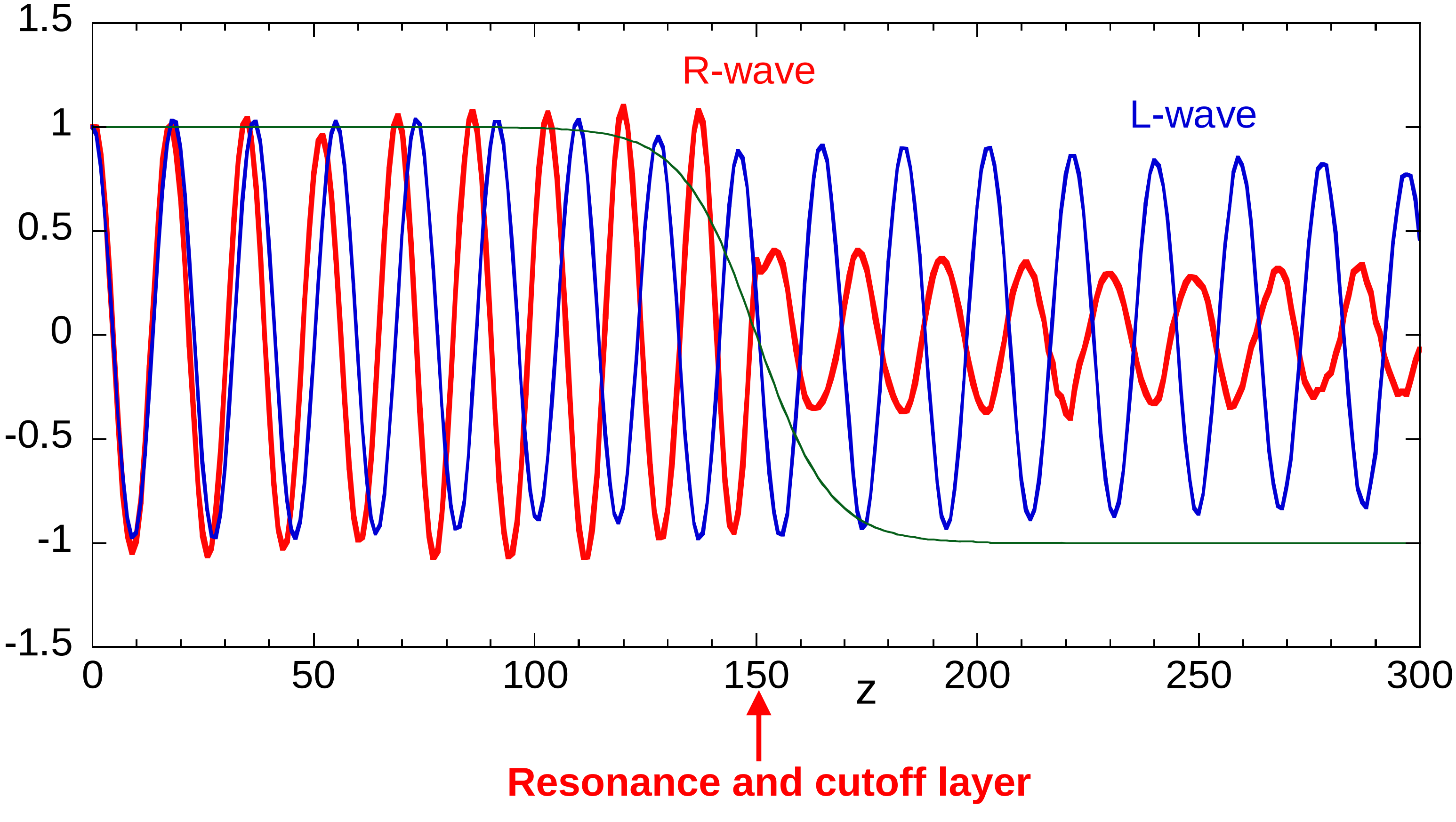
2-d simulation for fundamental ECRH
(Right-handed circularly polarized wave with ω = 28GHz)



Tunneling of right-handed circularly polarized mode

(2-d simulation)

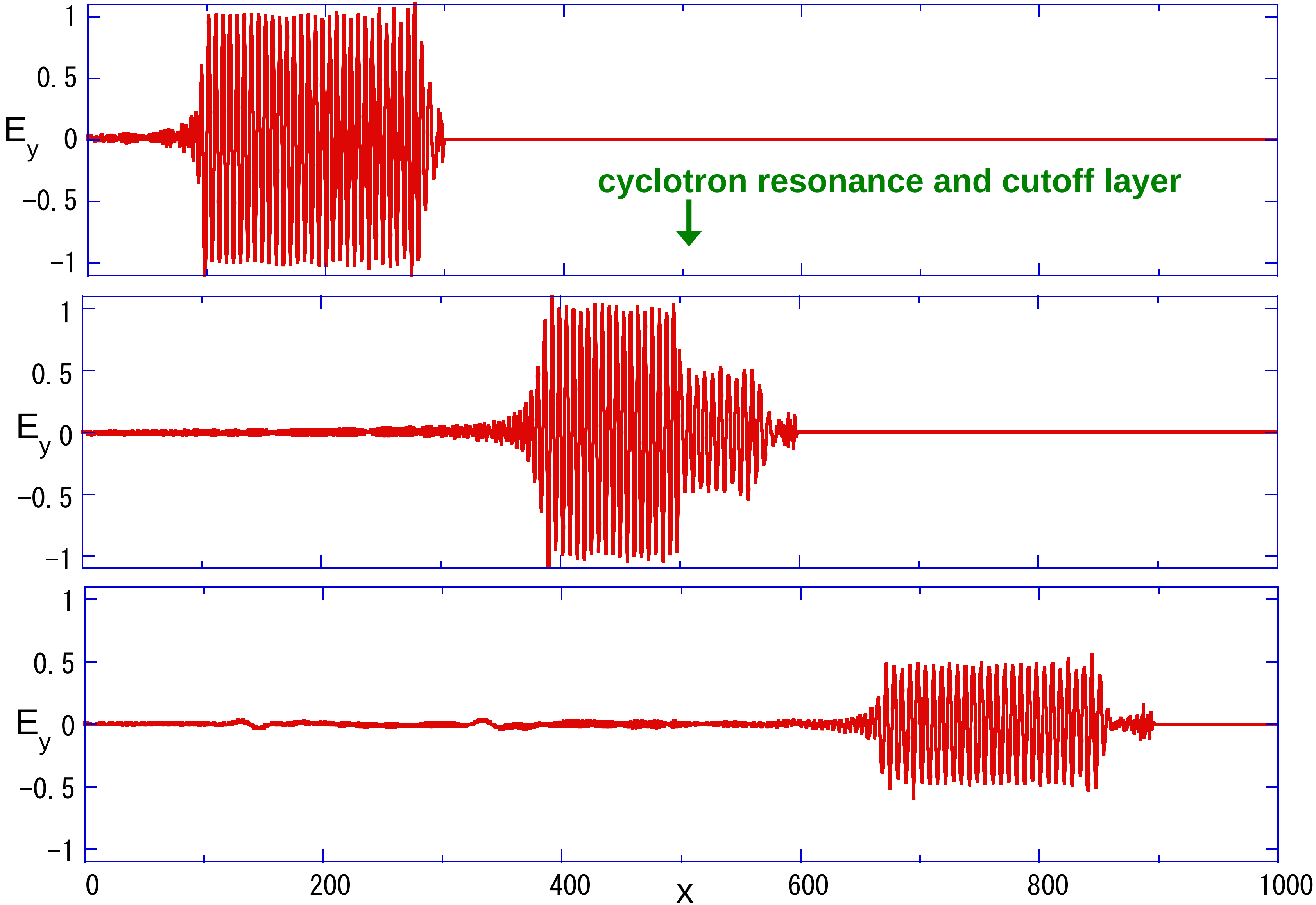
L-mode is not affected by resonance and cutoff layer.



Launched beam is not a plane wave, and then wave diffraction occurs.

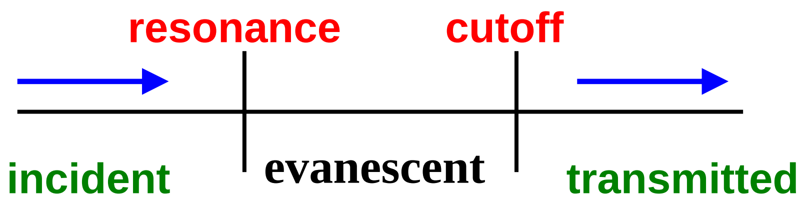
Tunneling of Right-handed circularly polarized mode

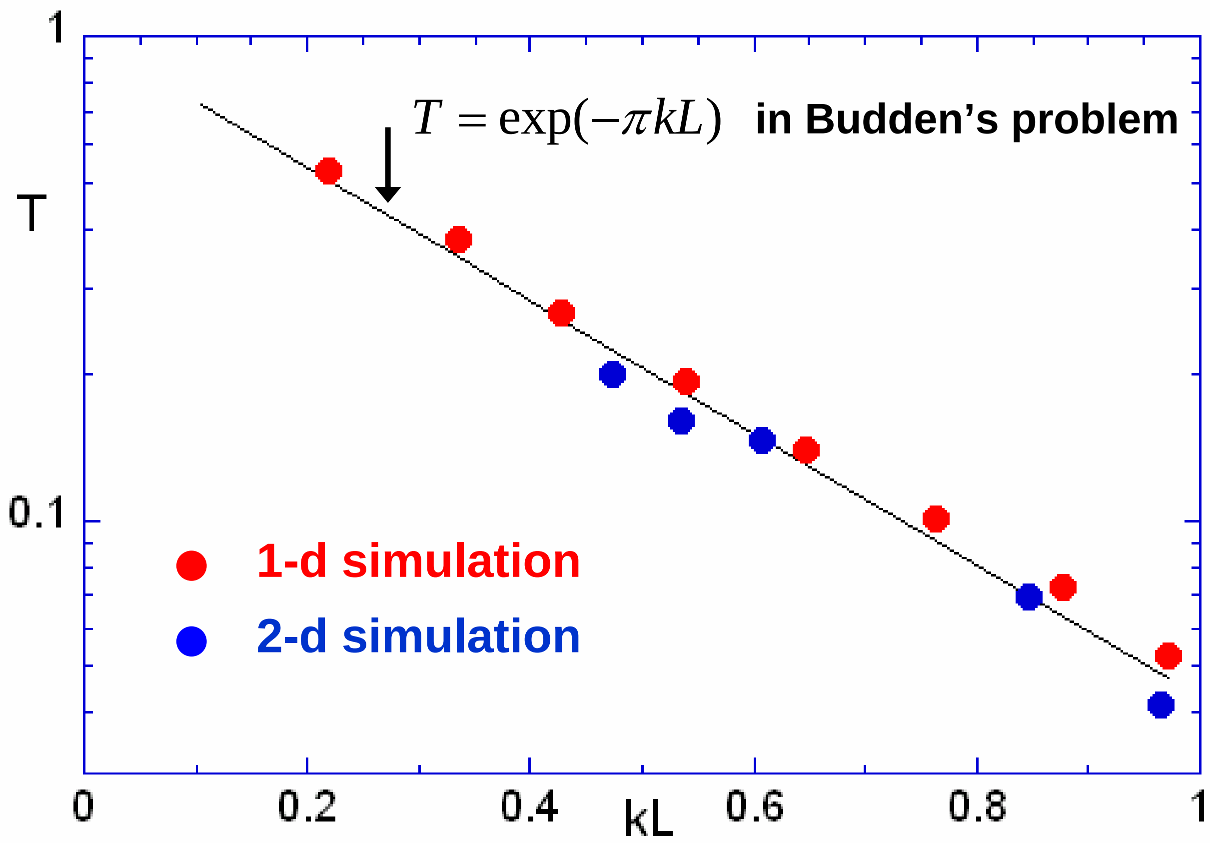
(1-d simulation)



Transmittance of right-handed circularly polarized mode

$$T = \frac{(\boldsymbol{E}\times\boldsymbol{B})_x\big|_{x=+\infty}}{(\boldsymbol{E}\times\boldsymbol{B})_x\big|_{x=-\infty}}$$





k = k(x = -∞)

Summary

We developed a Maxwell equation simulator, which approximately takes into account wave-particle interactions such as cyclotron resonance.

The code can treat wave diffraction, mode tunneling and also mode conversion of electromagnetic waves, in addition to estimate power deposition profile in ECRH.