Integration of Global Stability into the Simulation of a Burning Plasma Experiment

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CEMM

PPPL
PRINCETON PLASMA PHYSICS LABORATORY

The Center for Extended Magnetohydrodynamic Modeling

(Global Stability of Magnetic Fusion Devices)

a SciDAC activity…

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Outline

1. Vision of an Integrated Model of a Burning Plasma
2. Essential MHD Phenomena that needs to be modeled
3. Essential elements of a MHD model
4. Progress and status of 3D MHD modeling
5. Status of U.S. Initiative in Integrated Modeling of Burning Plasmas (FSP)

Present capability:

TSC (2D) simulation of an entire burning plasma tokamak discharge (FIRE)

Includes:

RF heating

Ohmic heating

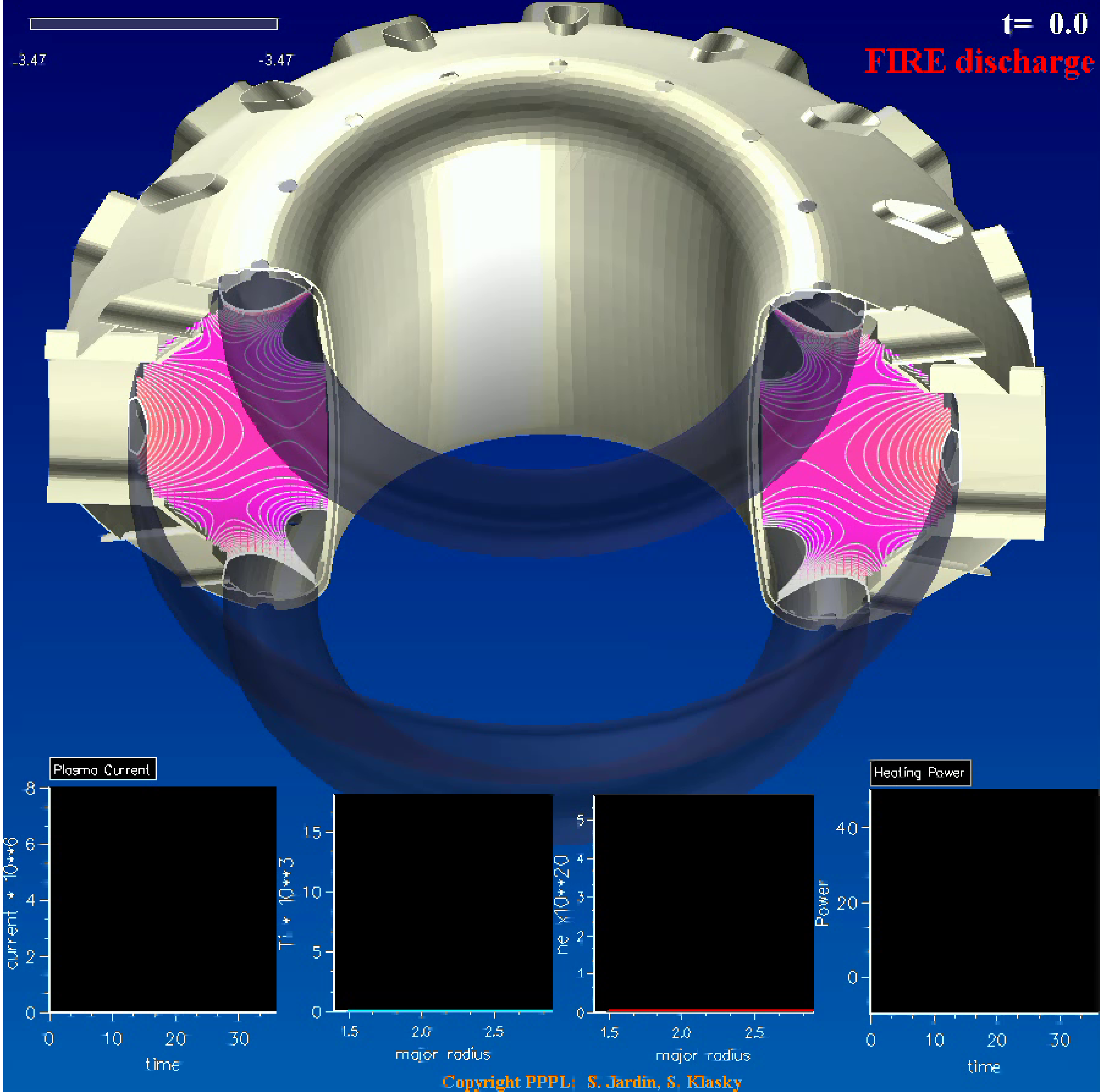
Alpha-heating

Microstability-based transport model

L/H mode transition

Sawtooth Model

Evolving Equilibrium with actual coils

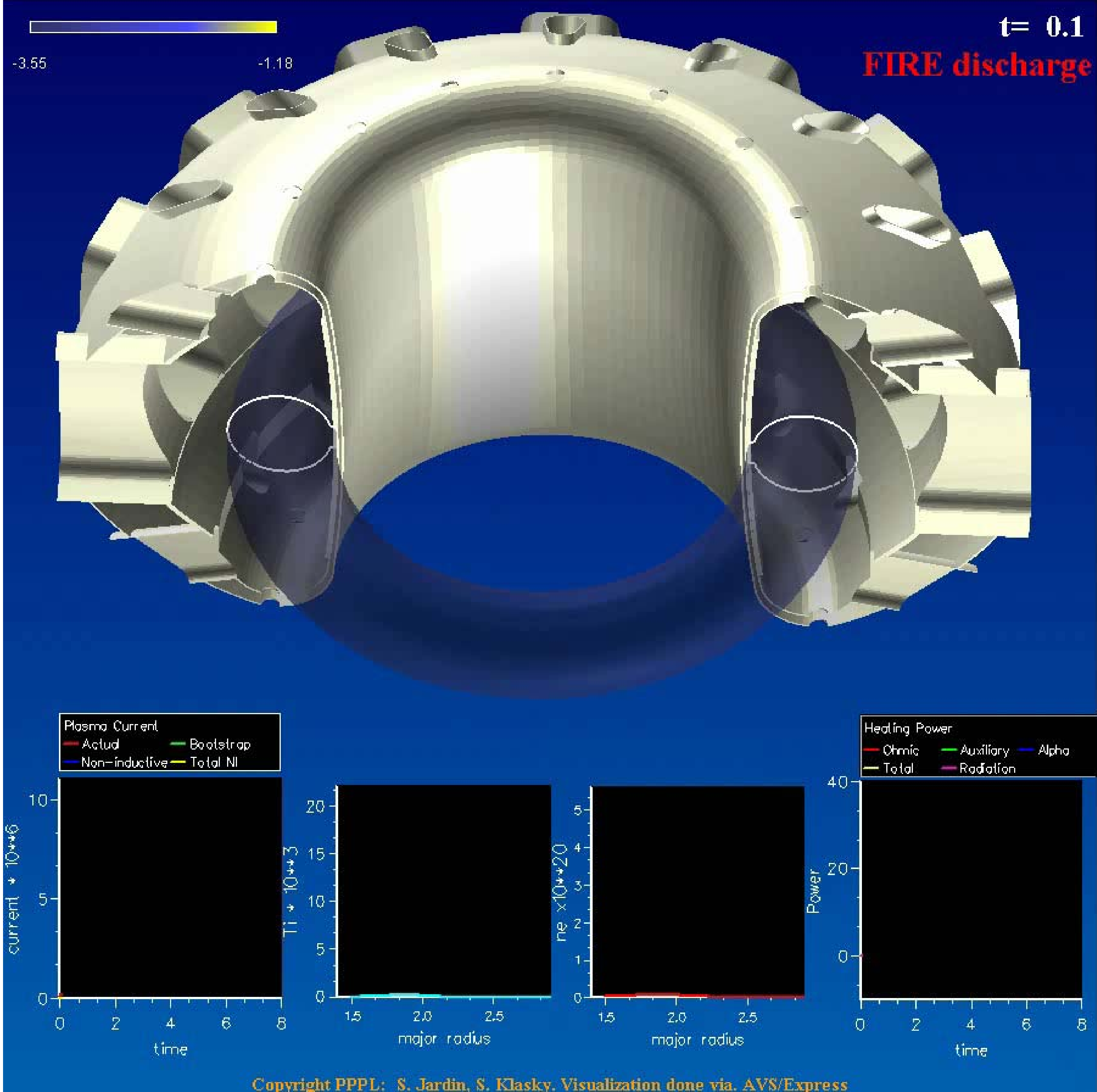


Even in 2D, things can go wrong:

Vertical Displacement Event (VDE) results from loss of vertical control due to sudden perturbation

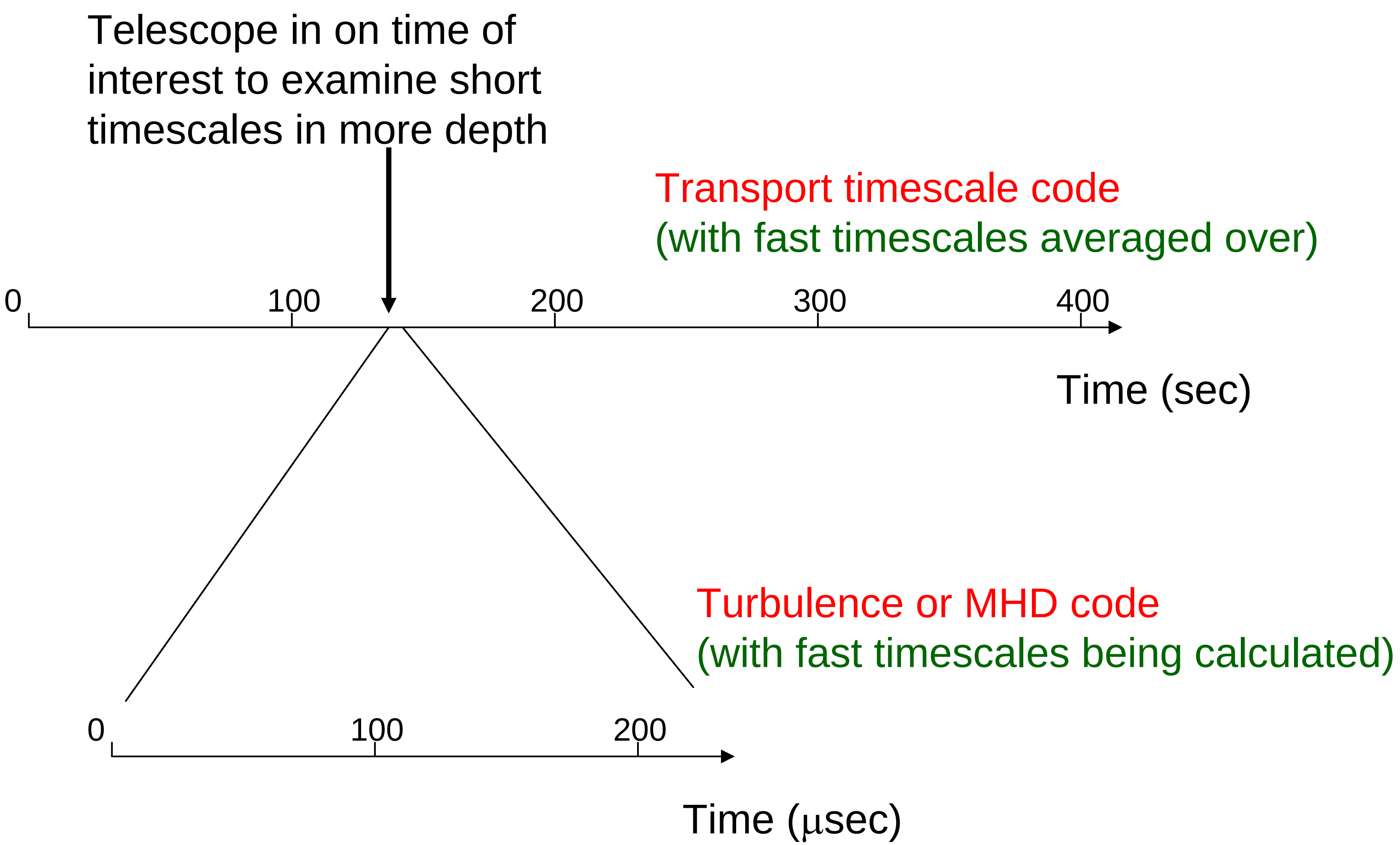
TSC simulation of an entire burning plasma discharge (FIRE)

Starts out same as before…ends in a VDE

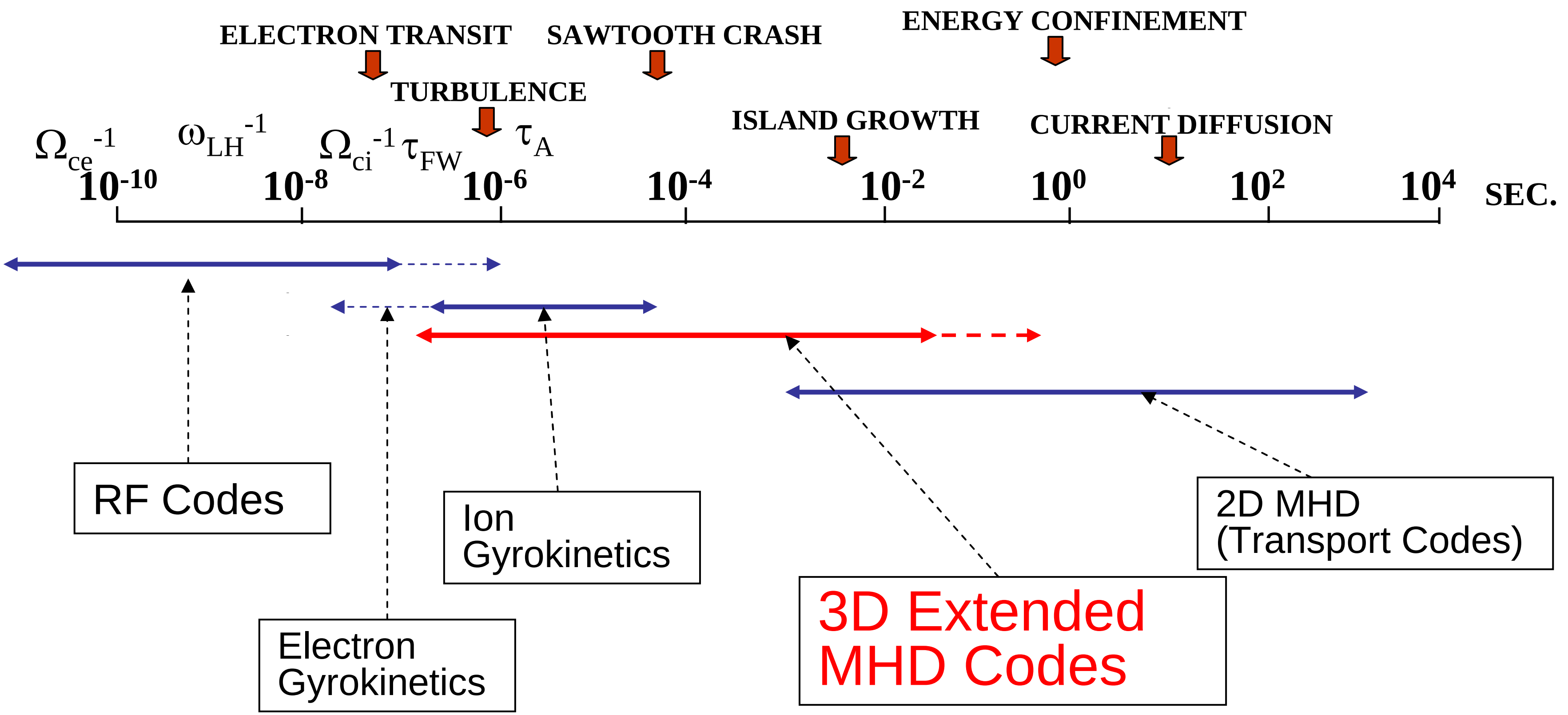


In 3D: Cannot solve for all phenomena with same set of equations:

In the foreseeable future, “integration” will mean looking at different timescale phenomena with different codes that talk to one another.

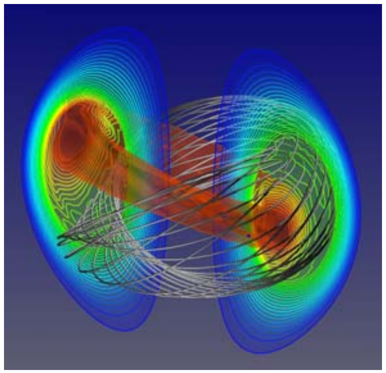


Telescoping in time is necessary because of the wide range of timescales present in a fusion device. Not possible to time-resolve all phenomena for entire discharge time as it would require 10^{12} or more time steps.

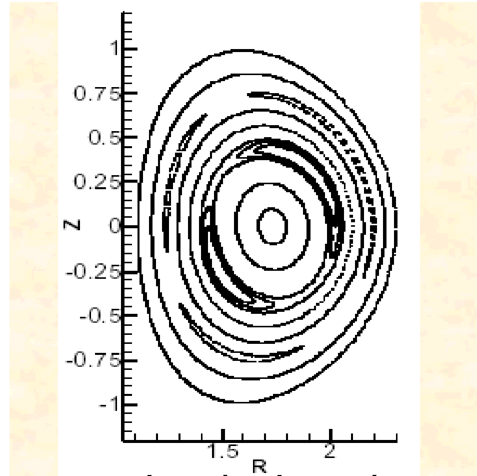


Time Scales in FIRE: B = 10 T, R = 2 m, $n_e = 10^{14}$ cm^{-3}, T = 10 keV

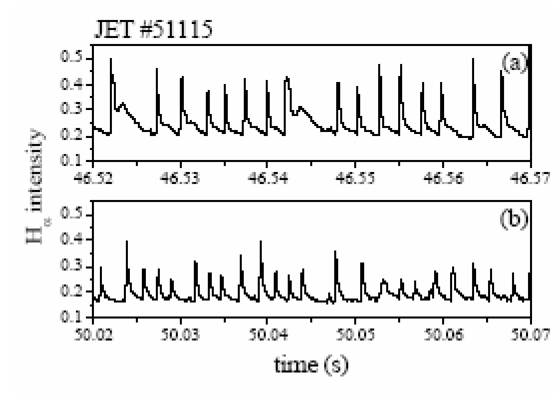
Essential MHD Phenomena that require Global 3D MHD Tokamak models



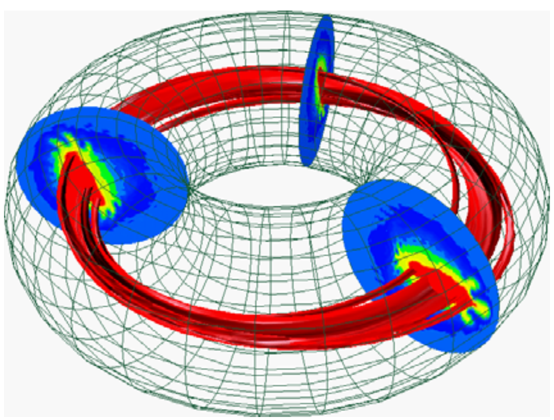
"sawtooth oscillations"



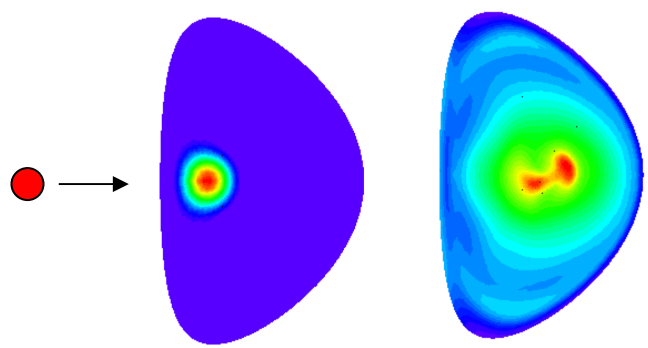
Neoclassical tearing modes and interaction of coupled island chains.



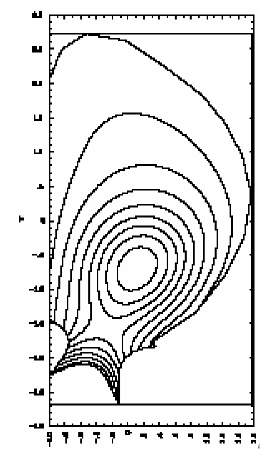
Edge Localized Modes



Disruptions caused by short wavelength modes interacting with helical structures.



Mass redistribution after pellet injection



Disruption forces and heat loads during VDE

Also, TAE modes, …

Plasma Models: XMHD

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\vec{E} + \vec{V} \times \vec{B} = \eta \vec{J} + \frac{1}{ne}\left[\vec{J} \times \vec{B} - \nabla \bullet P_e \right]$$

$$\mu_0 \vec{J} = \nabla \times \vec{B}$$

$$P = pI + \Pi$$

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V}\right) = \nabla \bullet P + \vec{J} \times \vec{B} + \mu \nabla^2 \vec{V}$$

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{V}) = S_M$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \nabla \bullet \left(\vec{q} + \frac{5}{2} P \bullet \vec{V} \right) = \vec{J} \bullet \vec{E} + S_E$$

$$\frac{3}{2}\frac{\partial p_e}{\partial t} + \nabla \bullet \left(\vec{q}_e + \frac{5}{2} P_e \bullet \vec{V}_e \right) = \vec{J} \bullet \vec{E} + S_E$$

Two-fluid XMHD: define closure relations for Π_i, Π_e, $\mathbf{q}_i$, $\mathbf{q}_e$

Hybrid particle/fluid XMHD: model ions with kinetic equations, electrons either fluid or by drift-kinetic equation

Difficulties in 3D MHD Modeling of Magnetic Fusion Experiments

Multiple timescales → Implicit methods and long running times

Multiple space-scales → Adaptive meshing, unstructured meshes, and implicit methods

Extreme anisotropy → High-order elements, field aligned coordinates, artificial field method

Essential kinetic effects → Hybrid particle/fluid methods, integrate along characteristics

CEMM Simulation Codes:

	NIMROD	M3D	AMRMHD*
Poloidal discritization	Quad and triangular high order finite elements	Triangular linear finite elements	Structured adaptive grid
Toroidal discritization	pseudospectral	Finite difference	Structured adaptive grid
Time integration	Semi-implicit	Partially implicit	Partially implicit and time adaptive
Enforcement of $\nabla \cdot \mathbf{B} = 0$	Divergence cleaning	Vector Potential	Projection Method
Libraries	AZTEC (Sandia)	PETSc (ANL)	CHOMBO (LBL)
Sparse Matrix Solver	Congugate Gradient	GMRES	Conjugate Gradient
Preconditioner	Line-Jacobi	Incomplete LU	Multigrid

*Exploratory project

NIMROD Time Advance: greater degree of implicitness

The **numerical formulation** is derived through the differential approximation for an implicit time advance for ideal linear MHD with arbitrary time centering, θ.

$$\rho\frac{\partial \mathbf{V}}{\partial t}-\theta\,\Delta t\left[\frac{1}{\mu_0}\left(\nabla\times\frac{\partial \mathbf{B}}{\partial t}\right)\times\mathbf{B}_0+\mathbf{J}_0\times\frac{\partial \mathbf{B}}{\partial t}-\nabla\frac{\partial p}{\partial t}\right]=\frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B}_0+\mathbf{J}_0\times\mathbf{B}-\nabla p$$

$$\frac{\partial \mathbf{B}}{\partial t}-\theta\,\Delta t\nabla\times\left(\frac{\partial \mathbf{V}}{\partial t}\times\mathbf{B}_0\right)=\nabla\times(\mathbf{V}\times\mathbf{B}_0)$$

$$\frac{\partial p}{\partial t}+\theta\,\Delta t\left(\frac{\partial \mathbf{V}}{\partial t}\cdot\nabla p_0+\gamma p_0\nabla\cdot\frac{\partial \mathbf{V}}{\partial t}\right)=-(\mathbf{V}\cdot\nabla p_0+\gamma p_0\nabla\cdot\mathbf{V})$$

Using the alternative differential approximation,

$$\rho\frac{\partial \mathbf{V}}{\partial t}-\theta^2\Delta t^2\mathbf{L}(\partial\mathbf{V}/\partial t)=\frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B}_0+\mathbf{J}_0\times\mathbf{B}-\nabla p+2\theta\Delta t\mathbf{L}(\mathbf{V})$$

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times(\mathbf{V}\times\mathbf{B}_0)$$

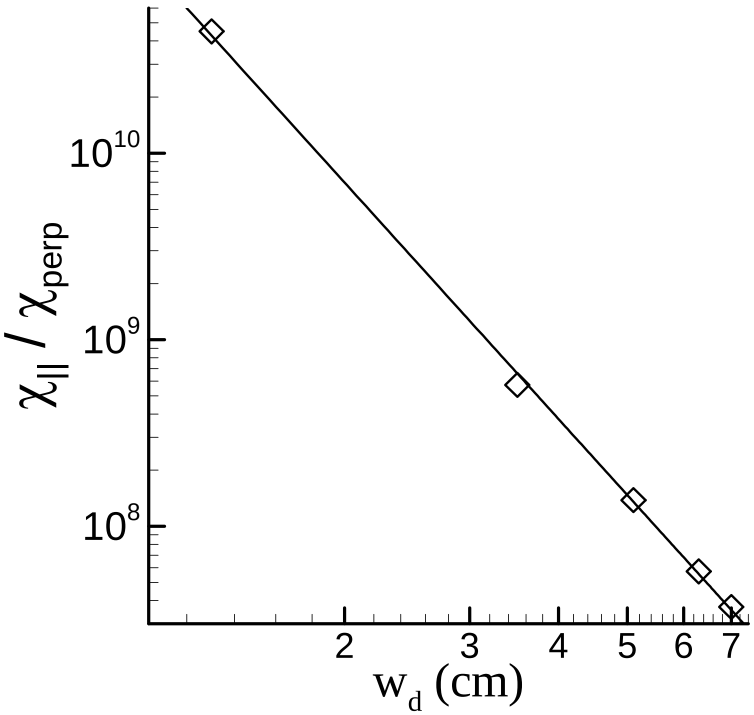
$$\frac{\partial p}{\partial t}=-(\mathbf{V}\cdot\nabla p_0+\gamma p_0\nabla\cdot\mathbf{V})$$

where $\mathbf{L}$ is the ideal MHD force operator. We may drop the Δt-term on the rhs to avoid numerical dissipation and arrive at a semi-implicit advance.

This approach requires solution of ill-conditioned linear systems at each step.

High order finite elements allows use of extreme values of thermal anisotropy.

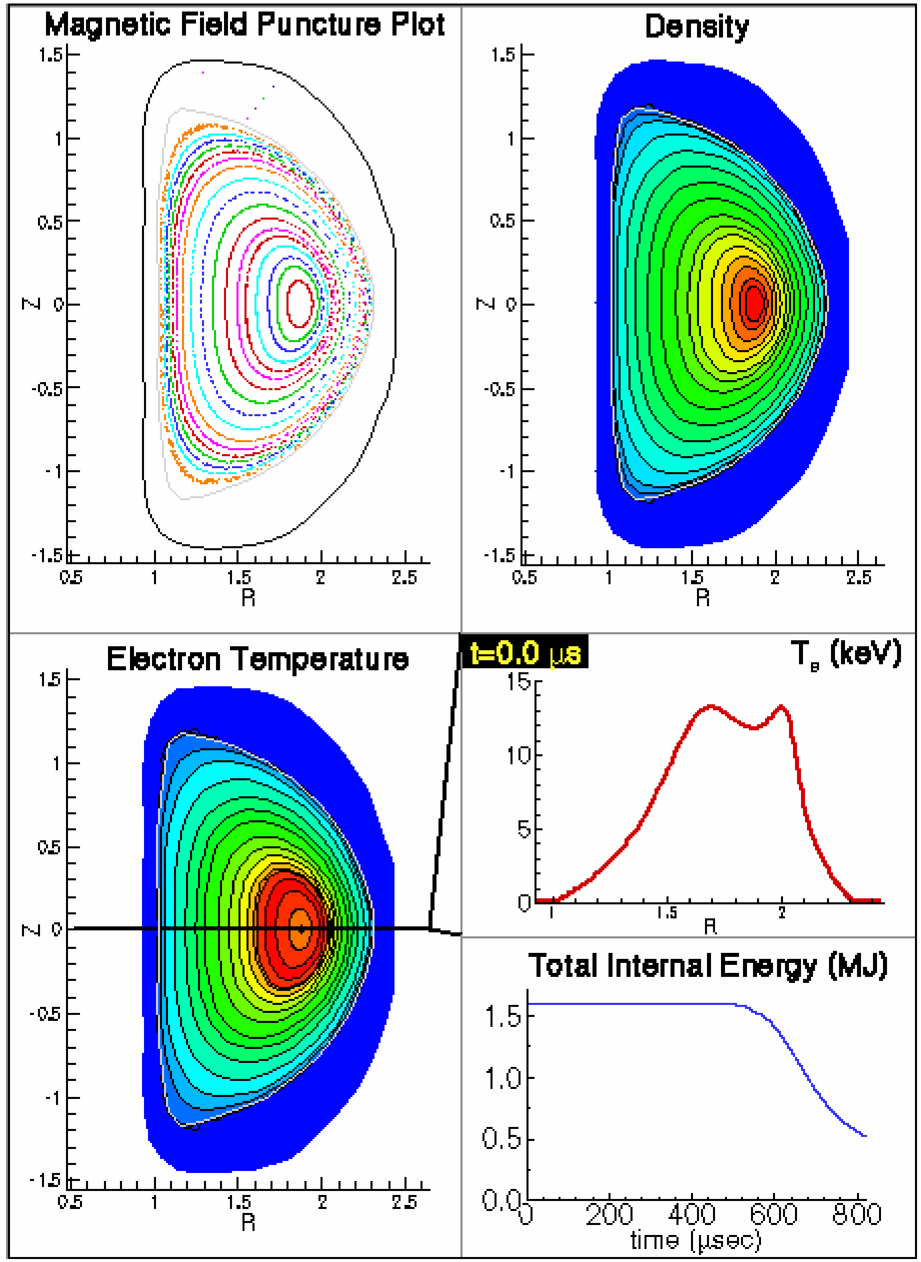
- 5th order accurate biquartic finite elements
- Repeat calculations with different conductivity ratios and observe effect on flattening island temperature
- Result extends previous analytic result to toroidal geometry.
- Implicit thermal conduction is required to handle stiffness.



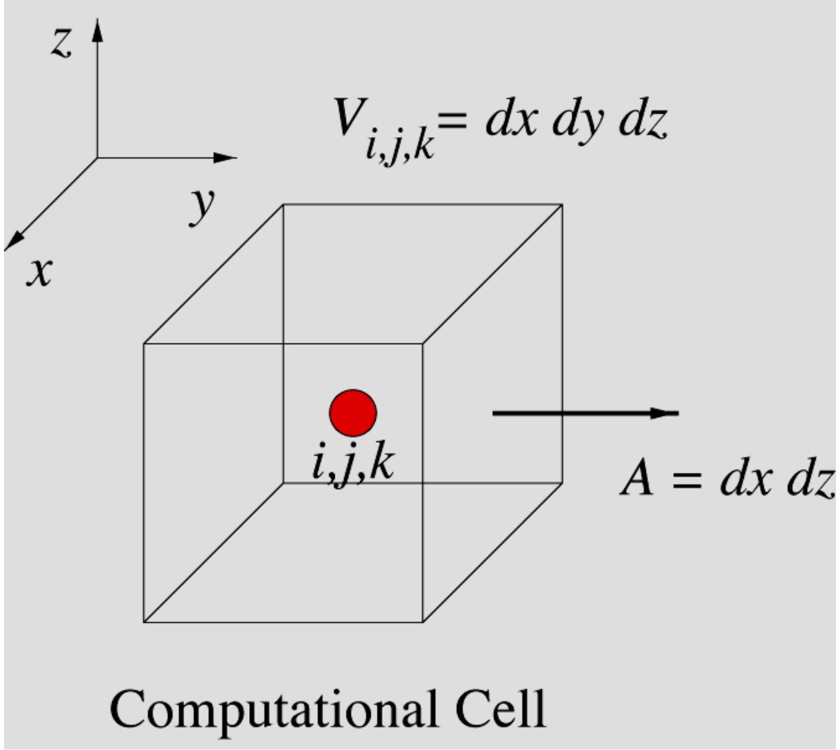
Example of a disruption thermal quench calculated by the NIMROD code. Plasma has been heated to exceed the ideal beta limit.

Thermal quench occurs due to field lines becoming stochastic, and parallel heat conduction can carry energy out of device.

Good qualitative agreement with DIII results



AMRMHD code uses Finite Volume approach



- Conservative (divergence) form of conservation laws:

$$\frac{dU}{dt} + \nabla \cdot F = S$$

- Volume integral for computational cell:

$$\frac{dU_{i,j,k}}{dt} = -\sum_{faces} A \cdot F + S_{i,j,k}$$

- Fluxes of mass, momentum, energy and magnetic field entering from one cell to another through cell interfaces.
- This is a Riemann problem.

Numerical Method in AMRMHD code

- Hyperbolic fluxes determined using the unsplit upwinding method (Colella, J. Comput. Phys., Vol 87, 1990)
  - Predictor-corrector (2^{nd} order in time)
  - Fluxes obtained by solving Riemann problem
  - Good phase error properties due to corner coupling terms

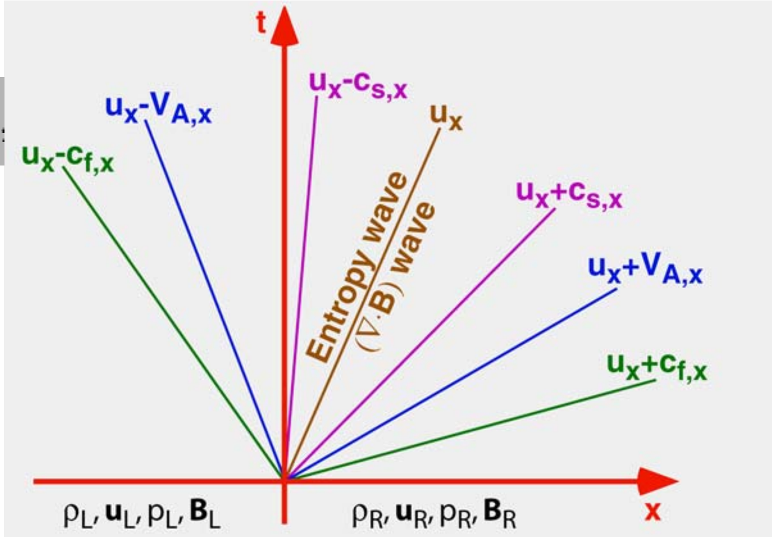
$$F^{n+\frac{1}{2}}_{i+\frac{1}{2}e^d} = R(W^{n+\frac{1}{2}}_{i,+,d}, W^{n+\frac{1}{2}}_{i+e^d,-,d}, d)$$

$$U^{n+1}_i = U^n_i - \frac{\Delta t}{h}\sum_{d=0}^{\mathbf{D}-1}\left(F^{n+\frac{1}{2}}_{i+\frac{1}{2}e^d} - F^{n+\frac{1}{2}}_{i-\frac{1}{2}e^d}\right)$$

- MHD Equations written in symmetrizable near-conservative form (Godunov, Numerical Methods for Mechanics of Continuum Media, 1, 1972, Powell et al., J. Comput. Phys., vol 154, 1999).

$$S_{\nabla\cdot\mathbf{B}}(U) = -\nabla\cdot\mathbf{B}(\{0, B_R, B_\phi, B_z, u_R, u_z, u_\phi, u_z, (B\cdot u)\}^T)$$

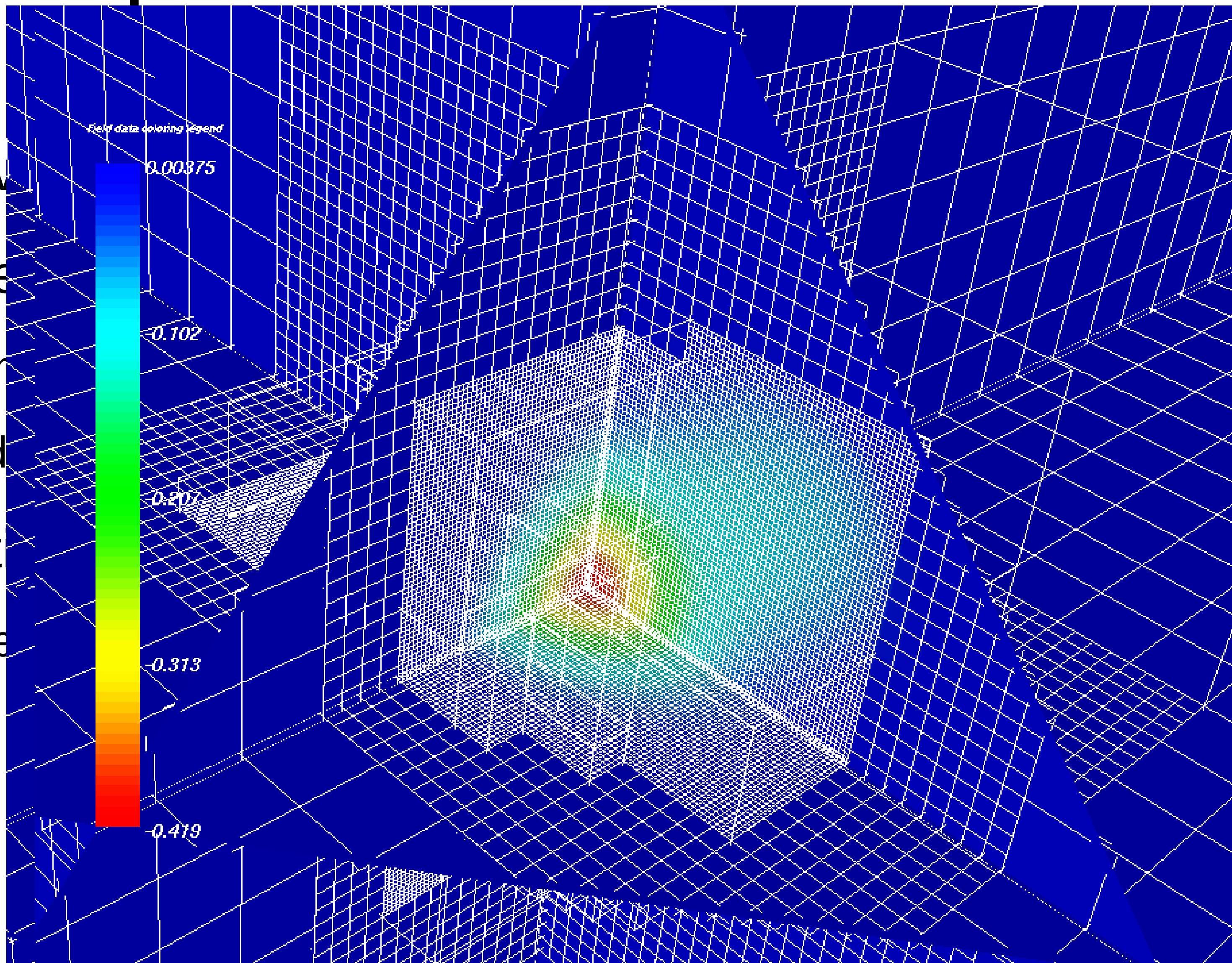
- The symmetrizable MHD equations lead to the 8-wave method.
  - The fluid velocity advects both the entropy and div(**B**)



Each eigenvector is treated in an upwind manner for it's eigenvalue

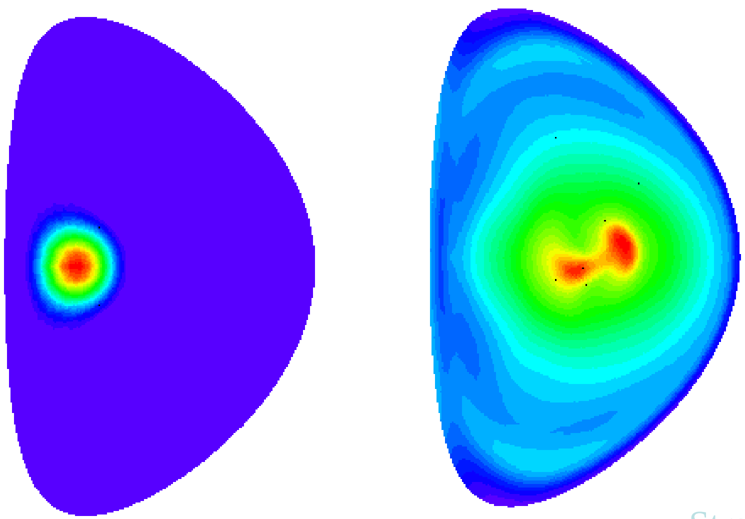
Adaptive Mesh Refinement

- Adaptiv e”
- Provide
  - Mesh
  - Lead
- Essenti
  - pelle



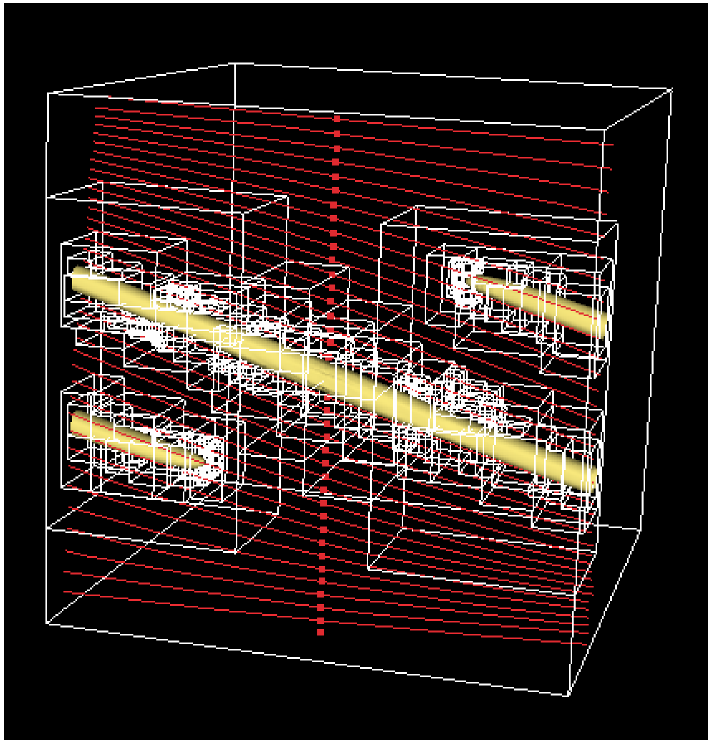
AMR technique is required to provide a quantitative description of pellet fueling of fusion plasmas

- Experimentally, it is known that injection of pellet can cause localized MHD instabilities that have large effect on fuelling efficiency,



Strauss/Park

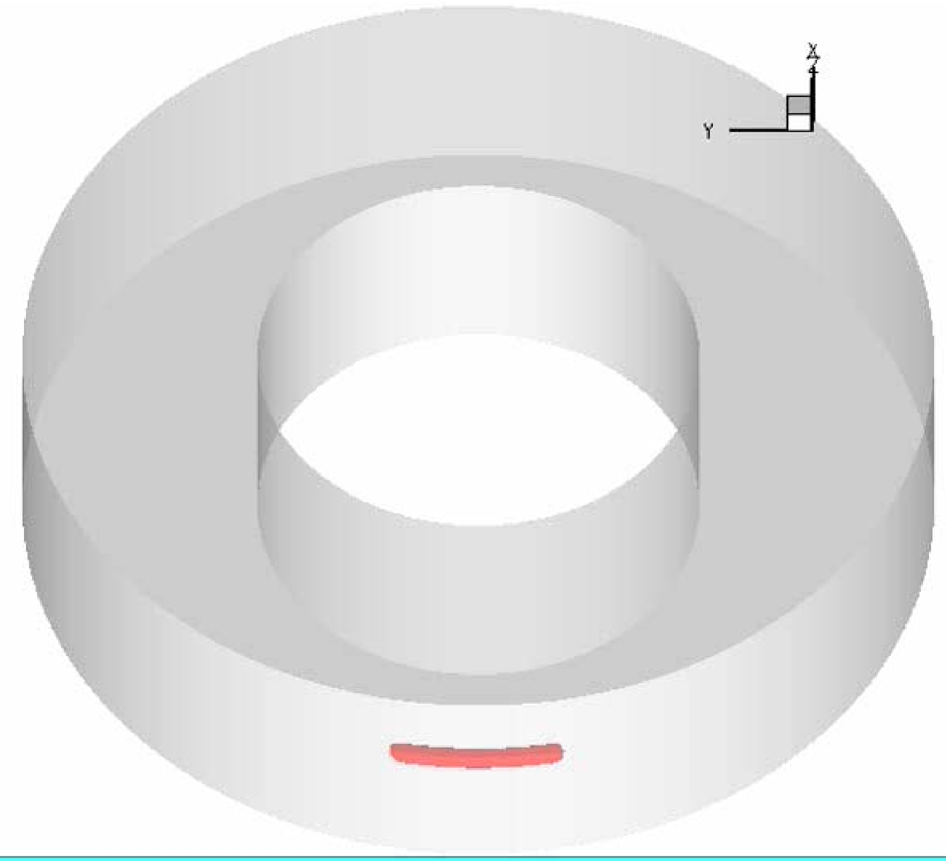
Initial M3D calculations (1998) showed essential physics, but at low resolution

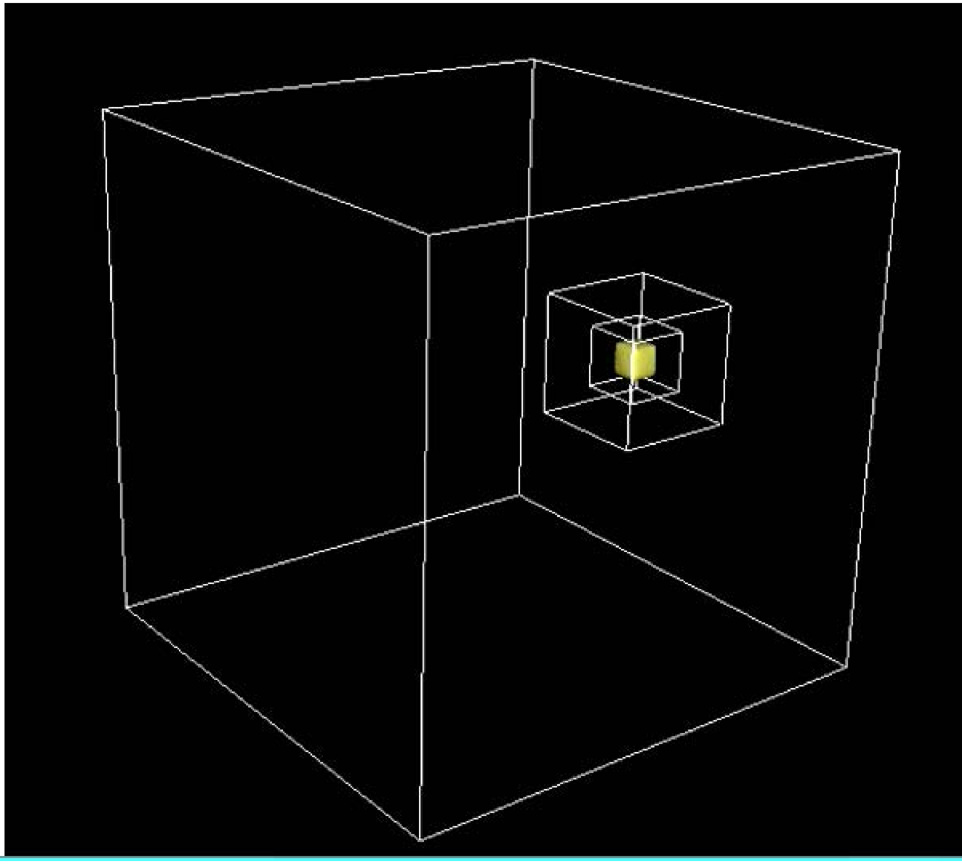


Initial AMR simulations of pellet injection in periodic cylinder illustrate that high resolution is possible; has now been extended to torus.

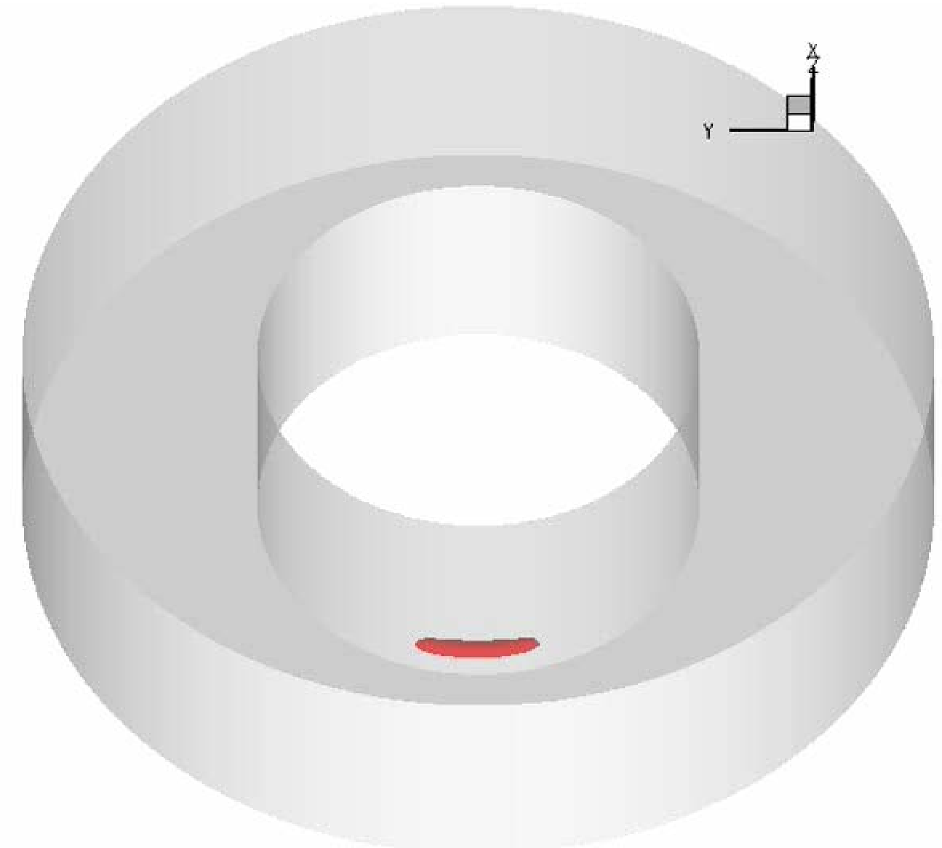
Samtaney

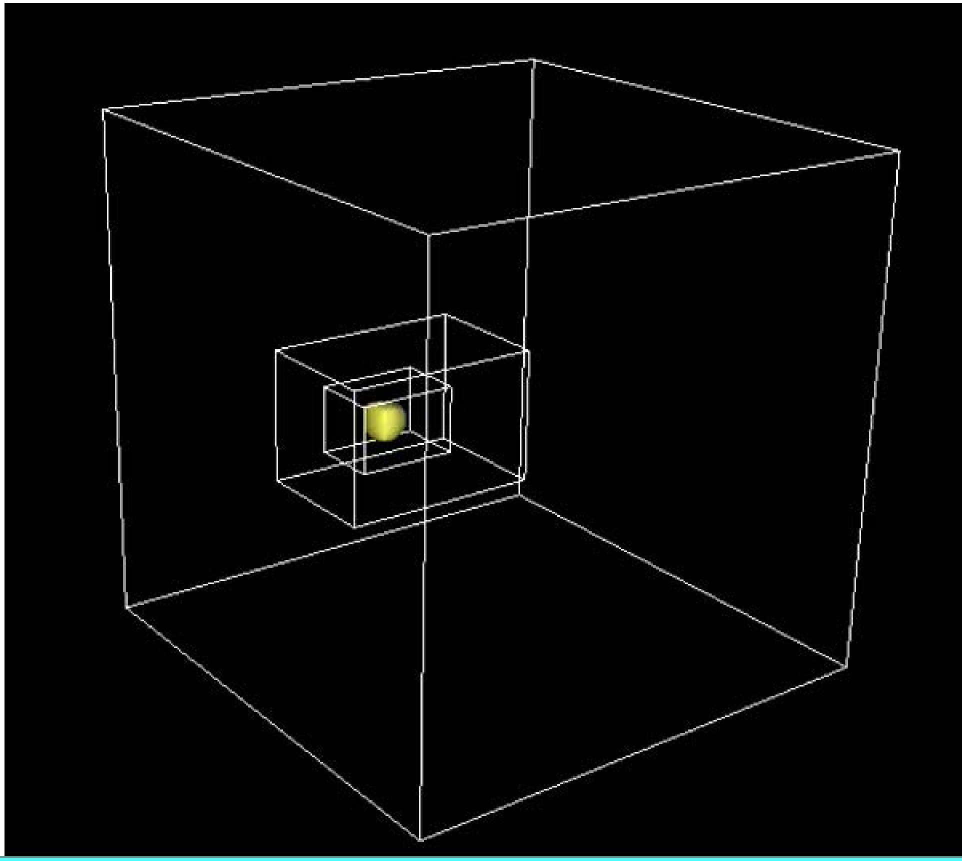
Low-field side pellet injection



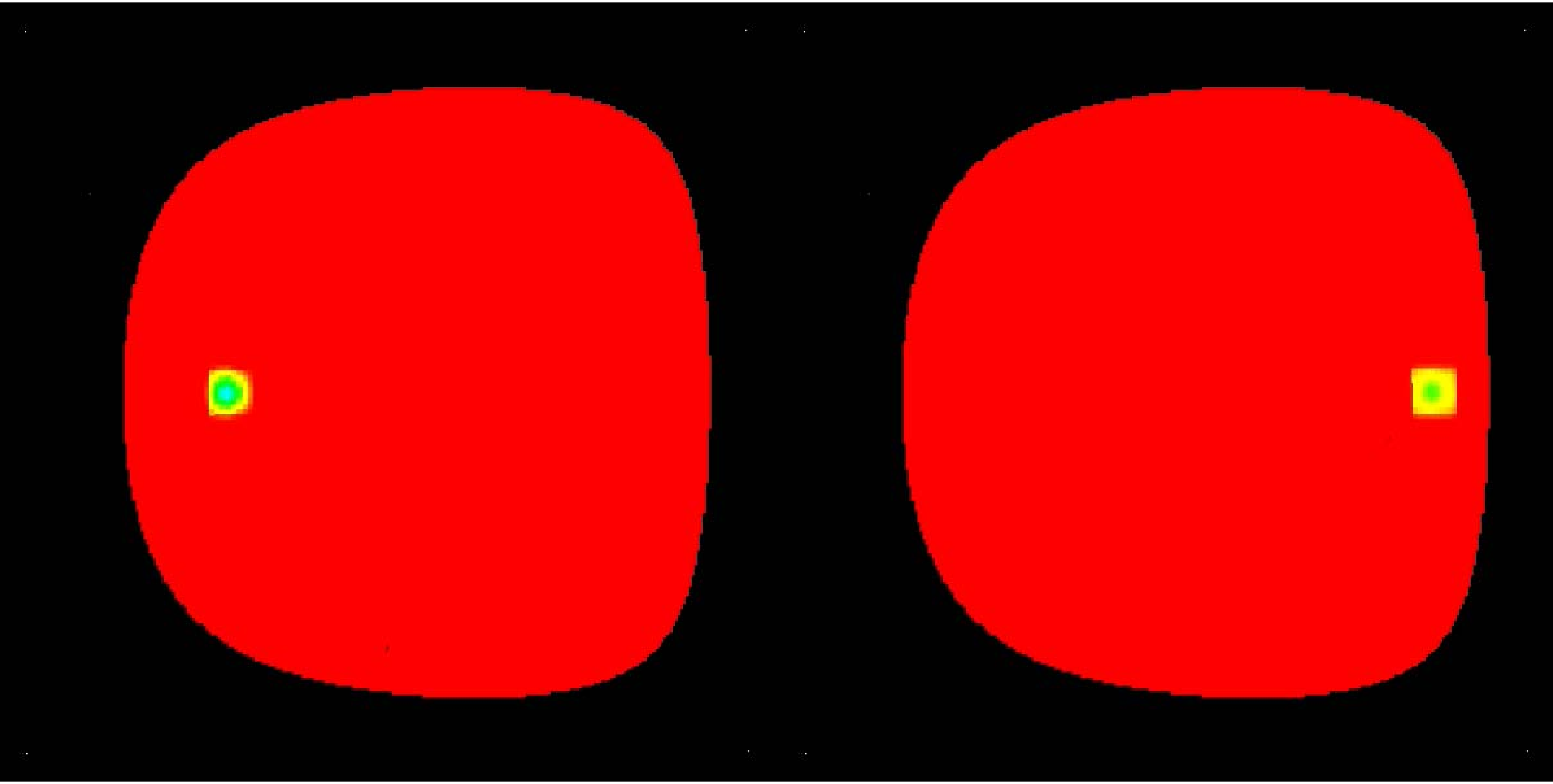


High-field side pellet injection





Comparison of LFS and HFS



Poloidal projection of density

Kinetic closures

M3D has Hybrid particle closure models

Field evolution equations are unchanged. Momentum equation replaced with "bulk fluid" and kinetic equations for energetic particles

$$\rho_b \frac{d\vec{V}_b}{dt} = -\nabla p_b - (\nabla \bullet \overleftrightarrow{P}_h)_\perp + \vec{J} \times \vec{B}$$

or

$$\rho_b \frac{d\vec{V}_b}{dt} = -\nabla p_b + [\frac{1}{\mu_0}(\nabla \times \vec{B}) - \vec{J}_h] \times \vec{B} + q_h \vec{V}_b \times \vec{B}$$

ions are particles obeying guiding center equations

$$\dot{\vec{X}} = \frac{1}{B}\left[\vec{B}^* U + \hat{b} \times (\mu \nabla B - \vec{E})\right],$$

$$\dot{U} = -\frac{1}{B}\vec{B}^* \bullet \left(\mu \nabla B - \frac{e}{m}\vec{E}\right),$$

$$\dot{\mu} = 0$$

$(\vec{X}, U, \mu)$ *are gyrocenter coordinates*

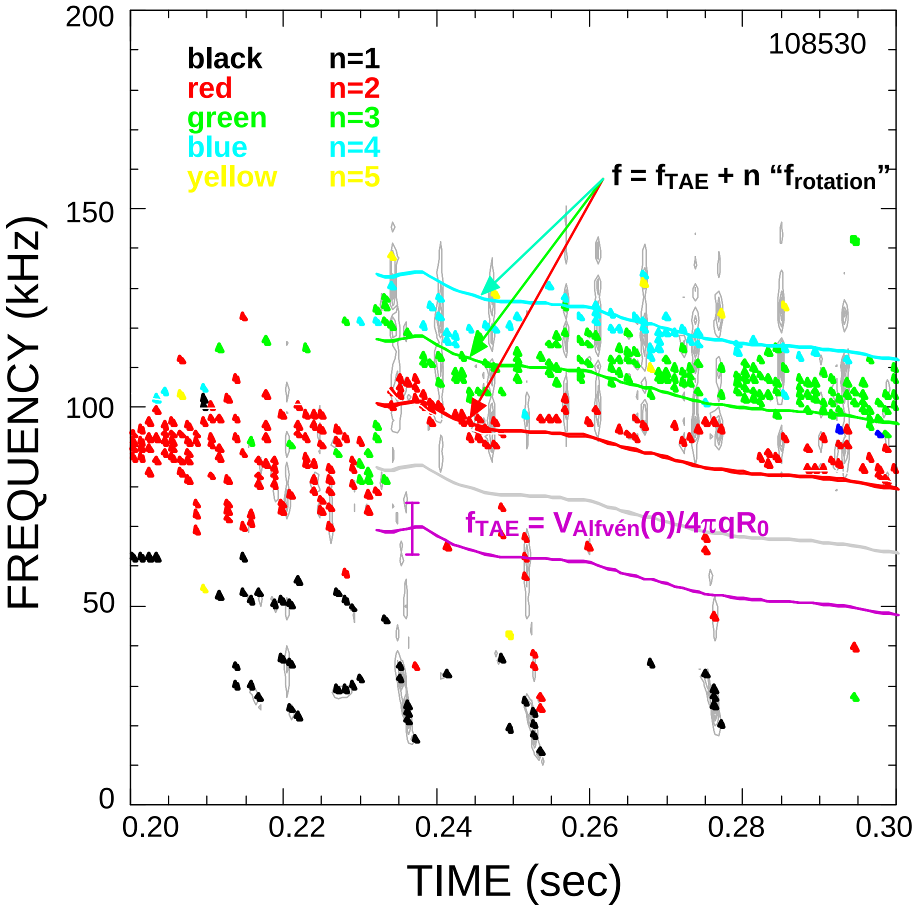
$$\vec{B}^* = \vec{B} + \frac{m}{e} U \hat{b} \times (\hat{b} \bullet \nabla \hat{b})$$

This hybrid model describes the nonlinear interaction of energetic particles with MHD waves

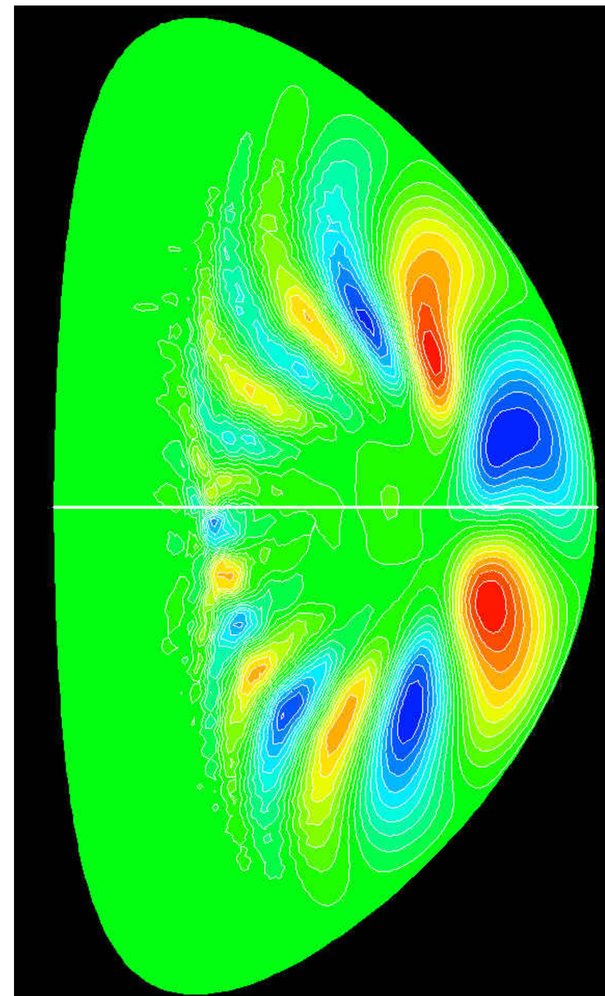
- small energetic to bulk ion density ratio
- 2 coupling schemes, pressure and current
- model includes nonlinear wave-particle resonances

Recent Application: Hybrid Simulations of unstable Toroidal Alfven Eigenmodes in NSTX

Computed frequencies are consistent with measurements for modes with toroidal mode numbers 1,2,3,4.



n=4 TAE



Example of a 3D calculation of an internal reconnection..

Or, Sawtooth event.

M3D simulation of NSTX

W. Park et al.

Visualization

S. Klasky et al.

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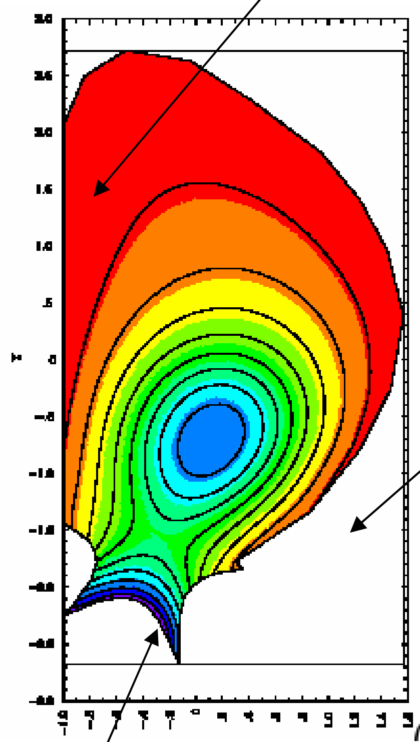
M3D Code now has thin shell and vacuum region in ITER geometry for calculation of non-axisymmetric VDE

Strauss, Pletzer, Park, Jardin, Breslau, Paccagnella

- Can now read initial equilibrium directly fromTSC
- Initial 3D simulations have been done
- Toroidal peaking factors as high as 3 have been observed
- Halo-current fraction transiently as high as 40%



In plasma:

$$\vec{B} = \nabla \psi \times \nabla \phi + \frac{1}{R} \nabla_{\perp} F + I \nabla \phi$$

$$\nabla \bullet \frac{1}{R} \nabla_{\perp} F = -\frac{1}{R^2} \frac{\partial I}{\partial \phi}$$

$$\nabla_{\perp} \equiv \nabla - \nabla \phi \bullet \frac{\partial}{\partial \phi}$$

In vacuum:

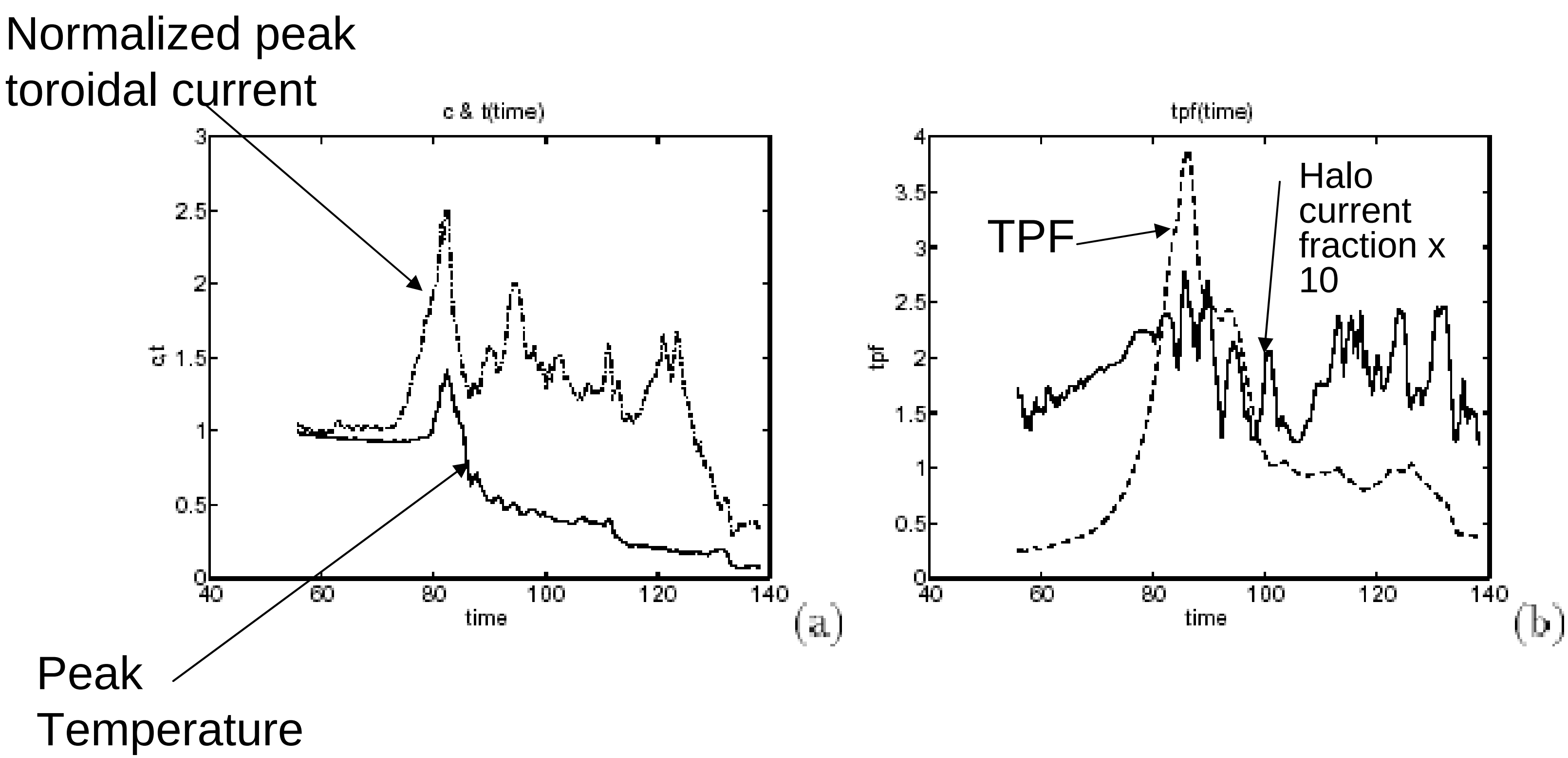
$$\vec{B}_V = \nabla \psi_V \times \nabla \phi + \nabla \lambda + I_0 \nabla \phi$$

$$\nabla \bullet \frac{1}{R^2} \nabla_{\perp} \psi_V = 0 \qquad \nabla^2 \lambda = 0$$

$$\frac{\partial \psi_V}{\partial \phi} = 0$$

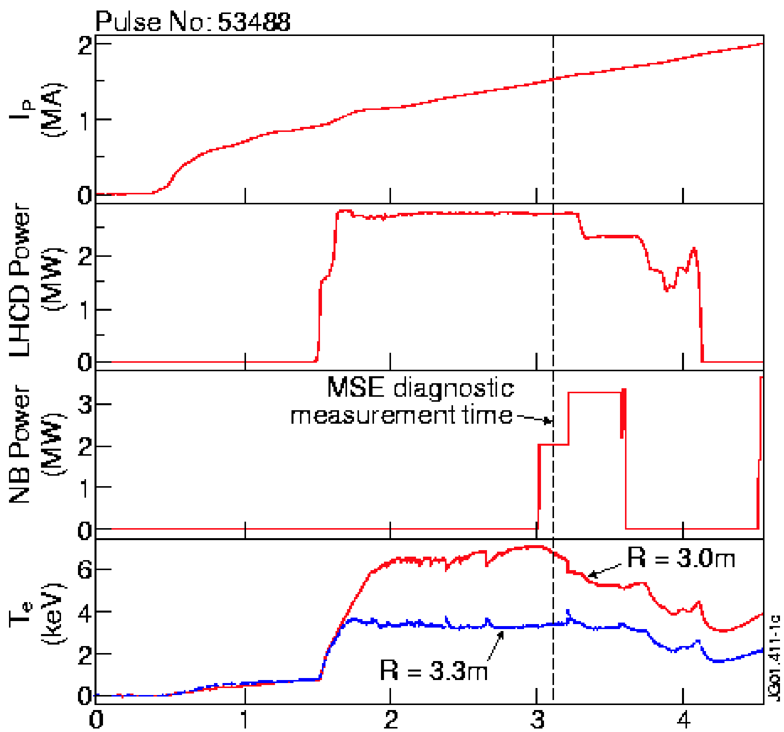
Thin Shell: $$\frac{\partial \psi}{\partial t} = \frac{\eta_W}{\delta} \left(\frac{\partial \psi_V}{\partial n} - R \frac{\partial \lambda}{\partial \ell} - \frac{\partial \psi}{\partial n} + \frac{\partial F}{\partial \ell} \right)$$

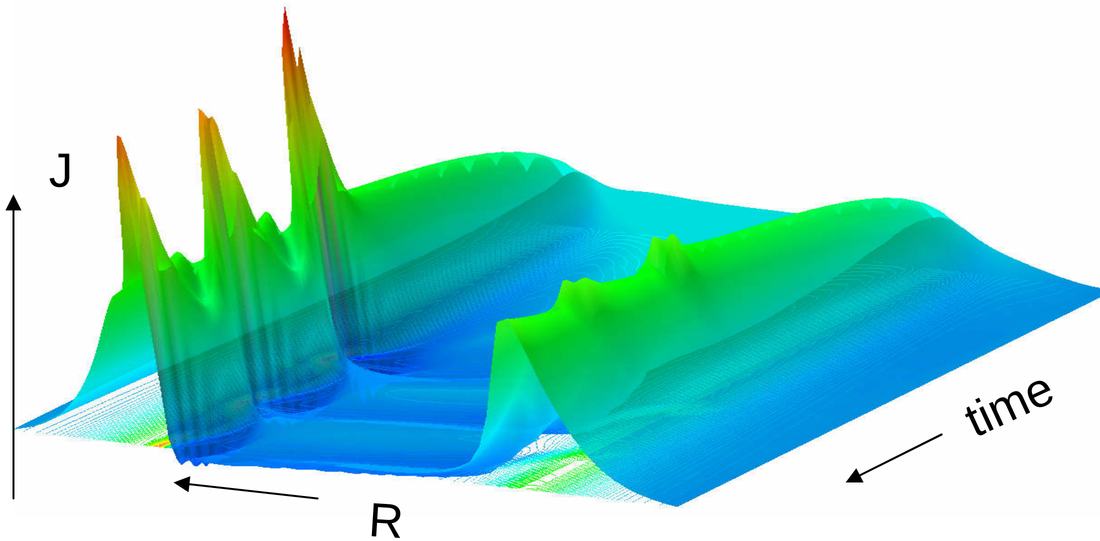
M3D 3D calculation of VDE in ITER



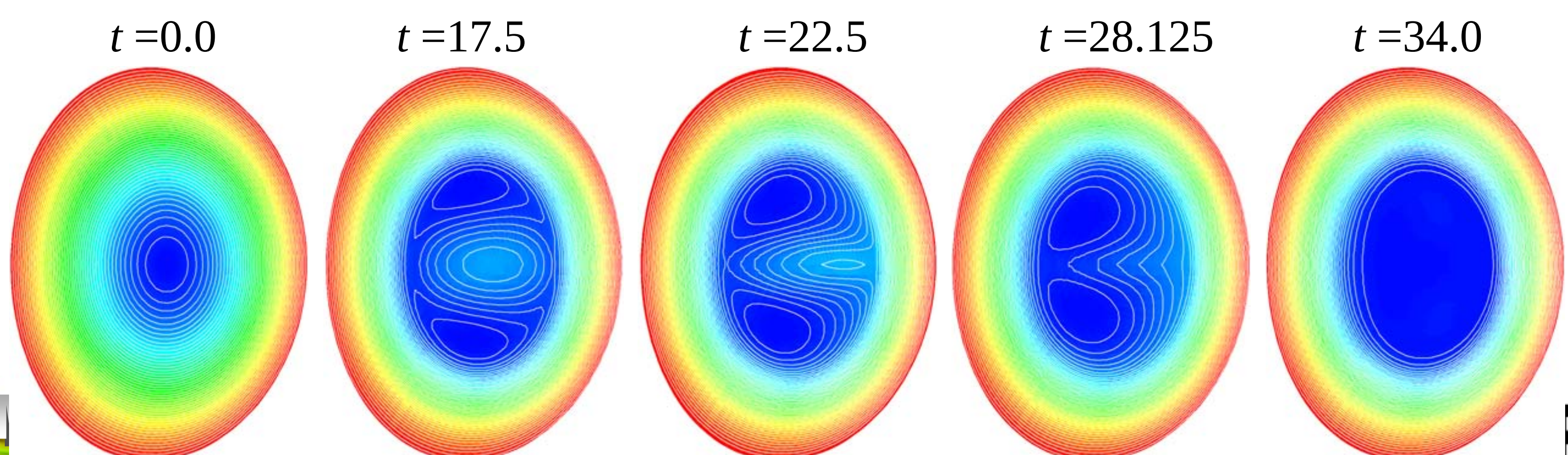
Preliminary results: Now starting calibration with TSC axisymmetric model

Recent Application: Interpretation of JET Current-Hole Experiments





Simulations have recently been extended to 2-fluid description and to finite β. Finite β island can cause reconnection to saturate, but rotation will destroy needed symmetry, and reconnection will result.



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Realistic simulation of a small tokamak: CDX-U :

Instead of modeling a big device for short times with unrealistic parameters, model a small device using the actual parameters:

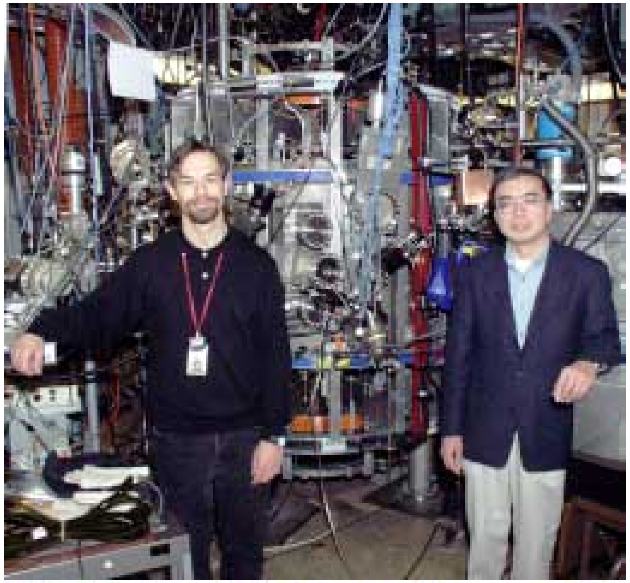


Fig 1: At the Current Drive Experiment Upgrade are Dick Majeski (left) and Bob Kaita, who co-headed the project.

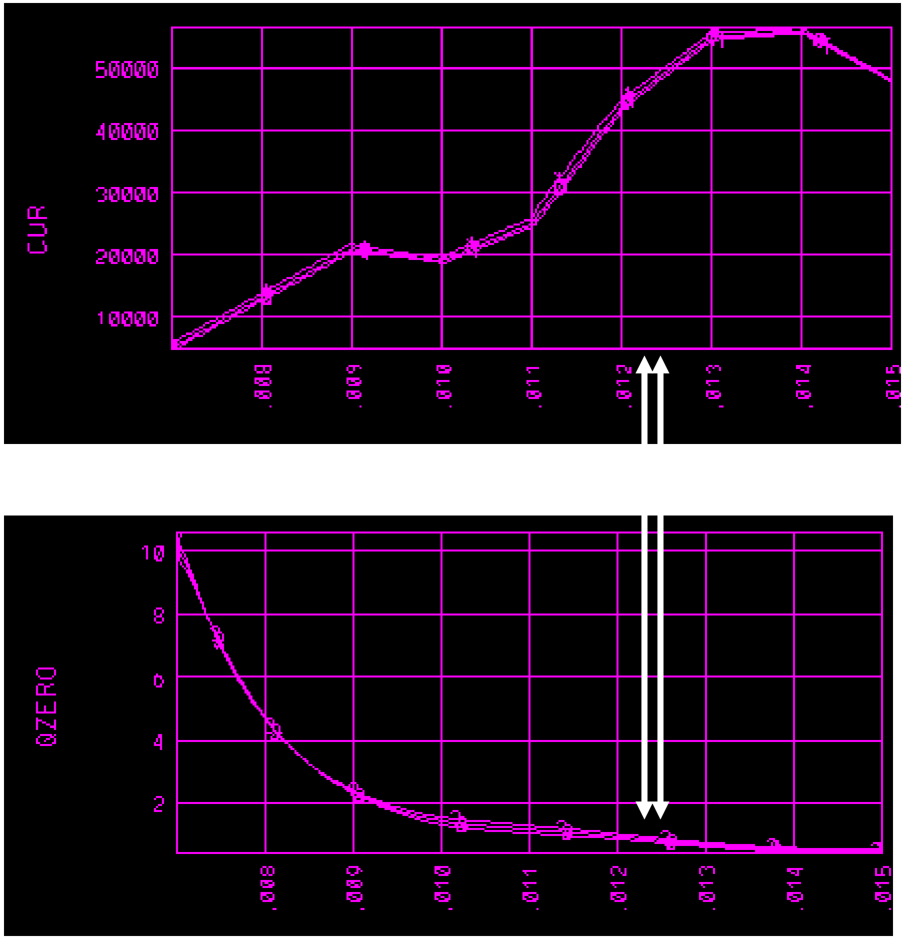
CDX-U Plasma Parameters		
Parameter	***Description***	***Value***
R_0	Major radius	33.5 cm
a	Minor radius	22.5 cm
$A=R_0/a$	Aspect ratio	1.5
κ	Plasma elongation	1.5-1.7
B_T	Toroidal magnetic field	2300 gauss
$n_e(0)$	Central electron density	$\sim 4\times10^{13}$ cm^{-3}
$T_e(0)$	Central electron temperature	100 eV
I_P	Plasma current	70 kA
	Pulse length	25 ms
	Pulse flat-top	5-10 ms

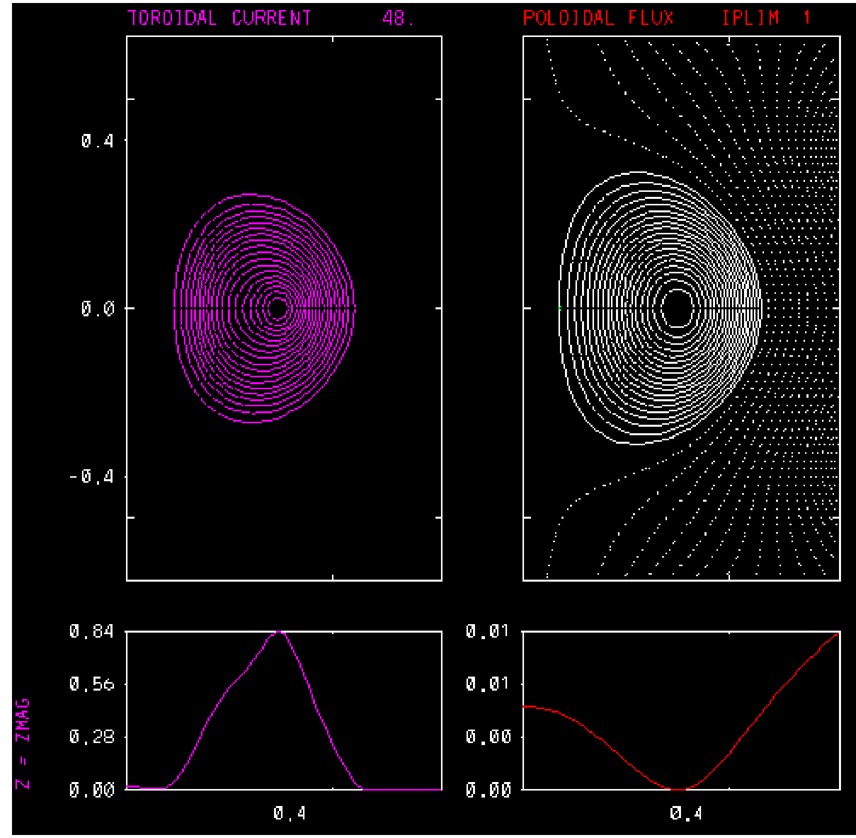
$(\rho^*)^{-1} = 40$ $\quad v_A = 10^8$ cm/sec $\quad T_{discharge} = .025$ ms $= 10^5\,\tau_A$

$S = 4 \times 10^4$ $\quad \tau_A = a/v_A = 2. \times 10^{-7}$ s $\quad$ PLT 10 Chord soft-X-ray

12 point Thompson

TSC follows 2D (axisymmetric) evolution of typical CDX-U discharge



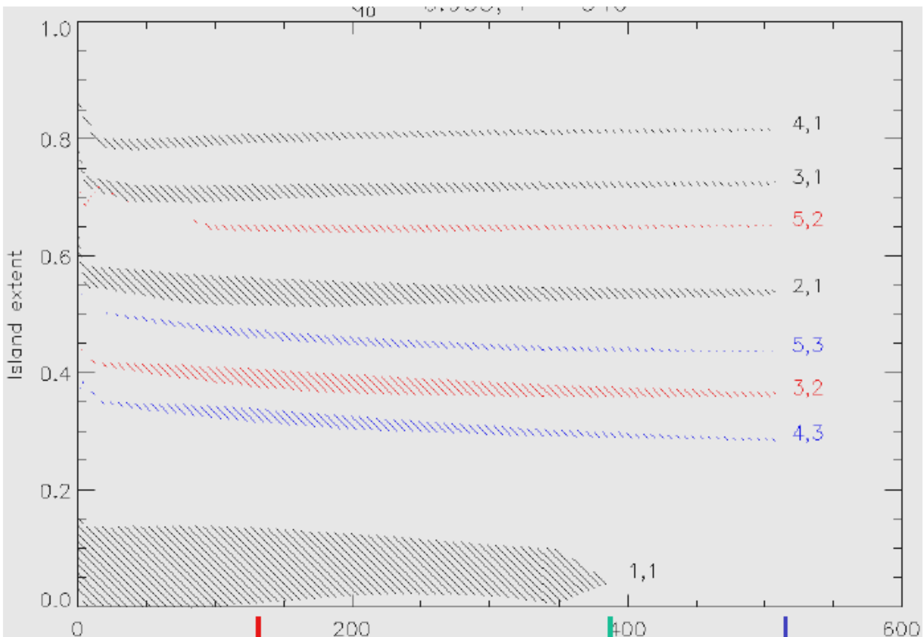


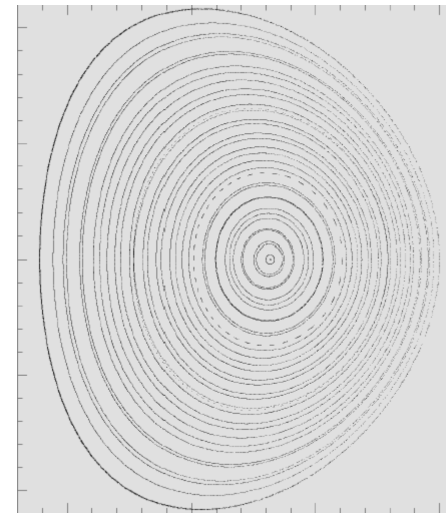
Equilibrium at t ~12.3ms (as q_0 drops to 0.95 or 0.89) is used to initialize 3D runs

M3D Resistive MHD: Magnetic Islands vs time for 2-initial conditions

CDX-U

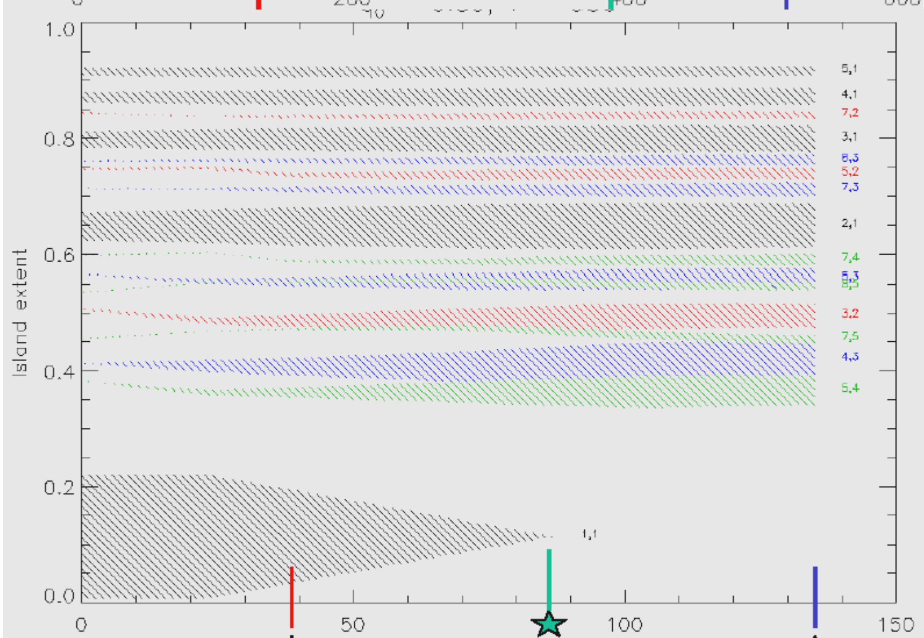
$q_0 = .95$

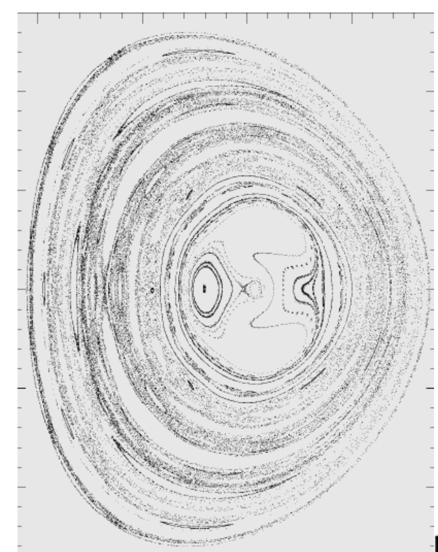




Restored axisymmetry

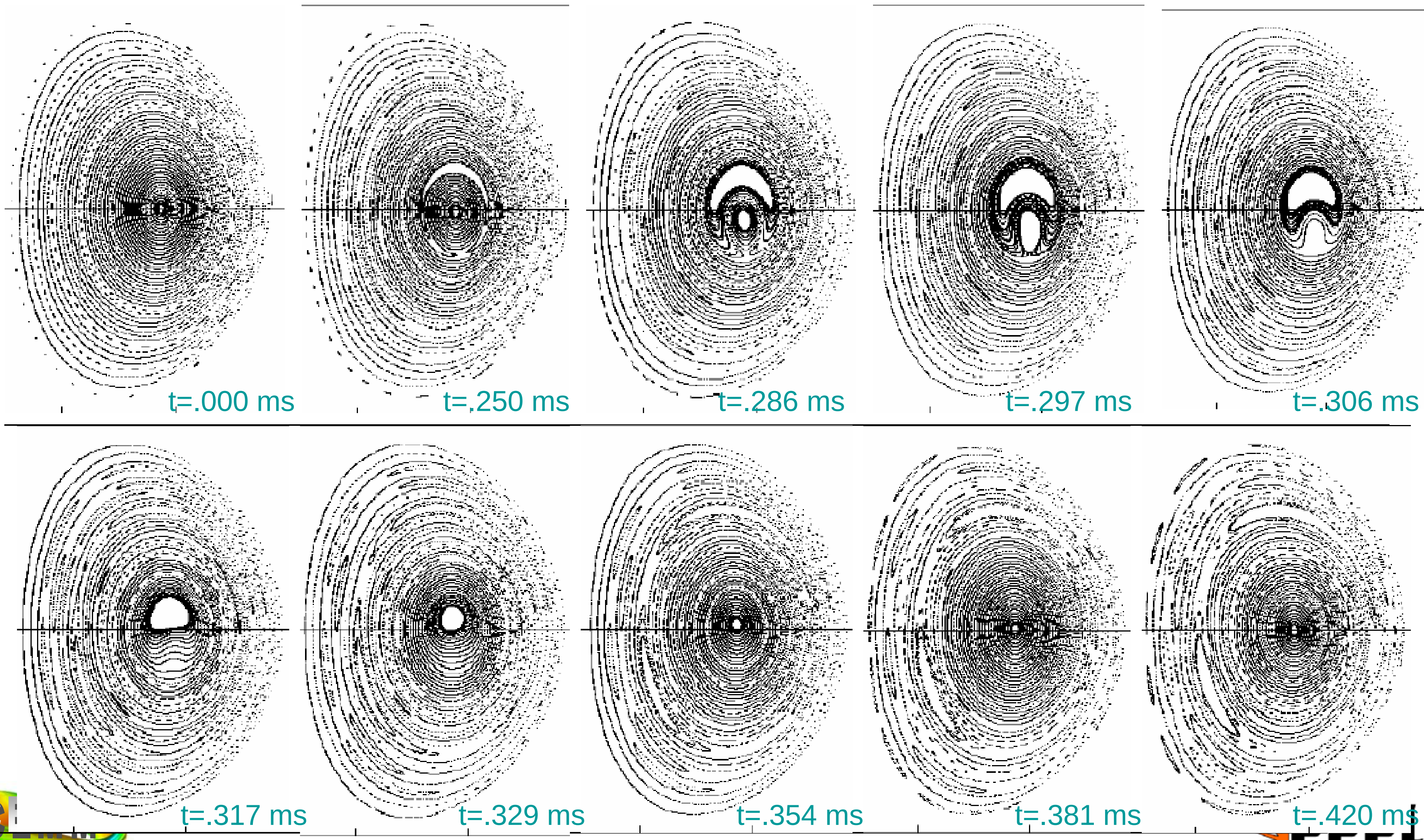
$q_0 = .89$





Disruption

Nimrod: Initial equilibrium with $q_0 = 0.95$



Required Resources

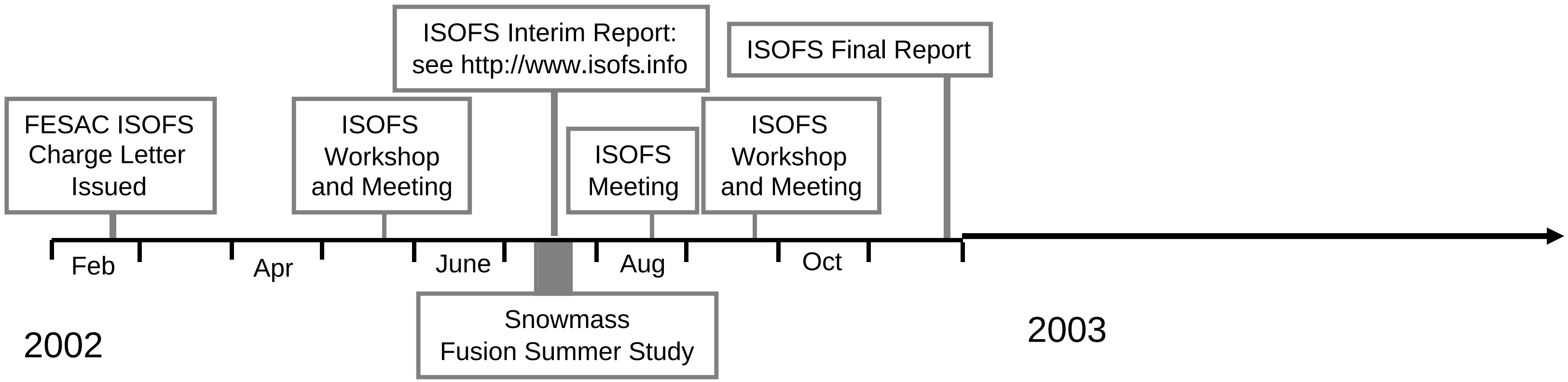
parameter	name	CDXU*	NSTX	CMOD	DIII-D	FIRE	ITER
R(m)	radius	0.3	0.8	0.6	1.6	2.0	5.0
Te[keV]	Elec Temp	0.1	1.0	2.0	2.0	10	10
β	beta	0.01	0.15	.02	0.04	0.02	0.02
$S^{1/2}$	Res. Len	200	2600	3000	6000	20000	60000
$(\rho^*)^{-1}$	Ion num	40	60	400	250	500	1200
$a/\lambda e$	skin depth	250	500	1000	1000	1500	3000
P	Space-time points	$\sim 10^{10}$	$\sim 10^{13}$	$\sim 10^{14}$	$\sim 10^{14}$	$\sim 10^{15}$	$\sim 10^{17}$

*Possible today

Estimate $P \sim S^{1/2} (a/\lambda e)^4$ for uniform grid explicit calculation. Adaptive grid refinement, implicit time stepping, and improved algorithms will reduce this.

Status of the US Initiative in Burning plasmas Modeling

- In Feb 2002, at the request of the Acting Director of the Office of Science, the Fusion Energy Science Subcommittee (FESAC) formed a subcommittee to look into Integrated Simulation of Fusion Systems (ISOFS)
- ISOFS FESAC subcommittee met during CY 2002, held 2 community-wide meetings, and submitted 2-volume report to FESAC in Dec 2002

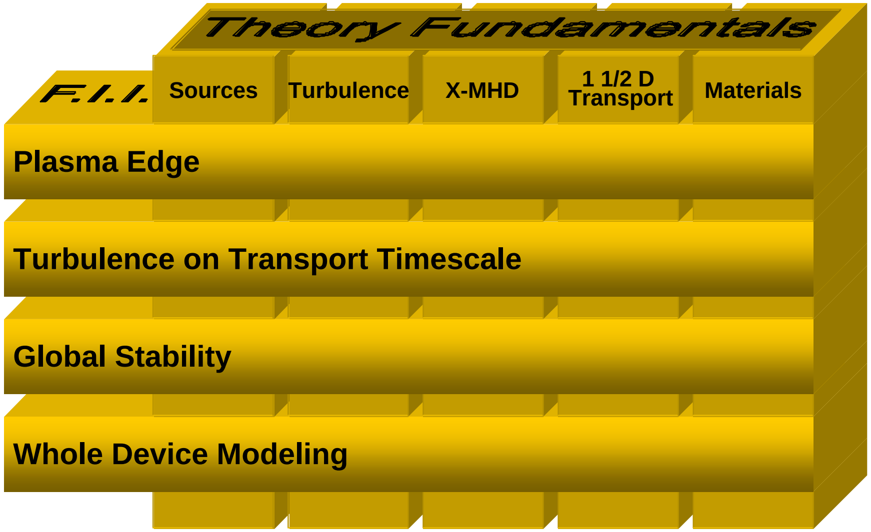


- DOE has now formed a steering committee to draft a management scheme and write a “call for proposals” : to be issued Dec 2004

The FII concept:

Focused Integration Initiatives are semi-autonomous working groups, each addressing one particular class of integration issues:

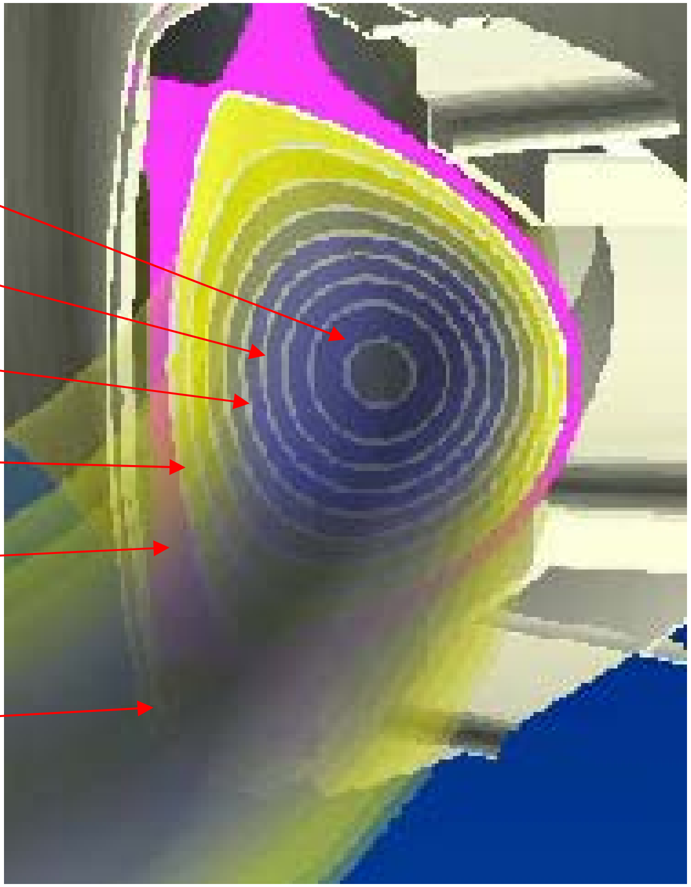
- decentralize management
- produce short-term scientific results of interest to the fusion program
- experiment with and gain experience with different framework paradigms



These will be chosen based on program balance and the degree to which compelling arguments can be made in the different areas.

Elements of an Integrated Tokamak Model

- Sawtooth region $q < 1$
  - (MHD and global stability)
- Core confinement region
  - (turbulent transport)
- Magnetic islands $q = 2$
  - (MHD and global stability)
- Edge pedestal region
  - (edge physics, MHD, turbulence)
- Scrape-off layer
  - (parallel flows, turbulence)
- Vacuum/Wall/Conductors/Antenna
  - MHD equilibrium, RF and NBI physics



Each of these different phenomena can be examined by an appropriate set of codes. Simplified models can be produced for use in the Whole Device Modeling code, and can be checked by detailed computation

Summary

1. “Integrated Model” needs to be able to telescope in on short time periods thought to be important to calculate nonlinear MHD events

2. These include sawteeth, ELMs, NTM, disruptions, pellet injection

3. MHD model needs to incorporate extreme anisotropy, multiple timescales, multiple spacescales, and kinetic effects

4. M3D, NIMROD, and AMRMHD codes have joined together in a CEMM initiative under the SciDAC program

5. U.S. Initiative in Integrated Modeling of Burning Plasmas (FSP) is now in the planning stages – based on FII concept.