EUROPEAN FUSION DEVELOPMENT AGREEMENT

REAL-TIME CONTROL OF INTERNAL TRANSPORT BARRIERS IN JET : EXPERIMENTS AND SIMULATIONS

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OUTLINE

1. Dimensionless ITB temperature or pressure gradient characterizing ITB's on JET, (also used on Tore Supra, FTU, Alcator C-Mod.

2. Technique for controlling the current and pressure profiles in high performance tokamak plasmas with ITB's : a technique which offers the potentiality of retaining the distributed character of the plasma parameter profiles.

3. First experiments using the simplest, lumped-parameter, version of this technique for the current profile :

   3.1. Control of the q-profile with one actuator : LHCD
   Modelling with CRONOS

   3.2. Control of the q-profile with three actuators : LHCD, NBI, ICRH
   Modelling with JETTO

*** PRELIMINARY***

4. Ongoing experiments on simultaneous real-time control of the current density + temperature gradient profiles.

Challenges of advanced profile control

Early experiments on JET were based on scalar measurements characterising the profiles (ρ_T*max) and/or other global parameters (l_i)

HOWEVER

1. ITB = pressure and current (+ rotation ...) **profiles**

Multiple-input multiple-output distributed parameter system (MIMO + DPS)

2. **Nonlinear interaction** between p(r) and j(r)

Feedback loop interaction

Need more information on the space-time structure of the system

Identify a high-order operator model around the target steady state

and try model-based DPS control using SVD techniques

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ITB dimensionless gradient criterion

Stabilisation of drift wave microturbulence through flow shear

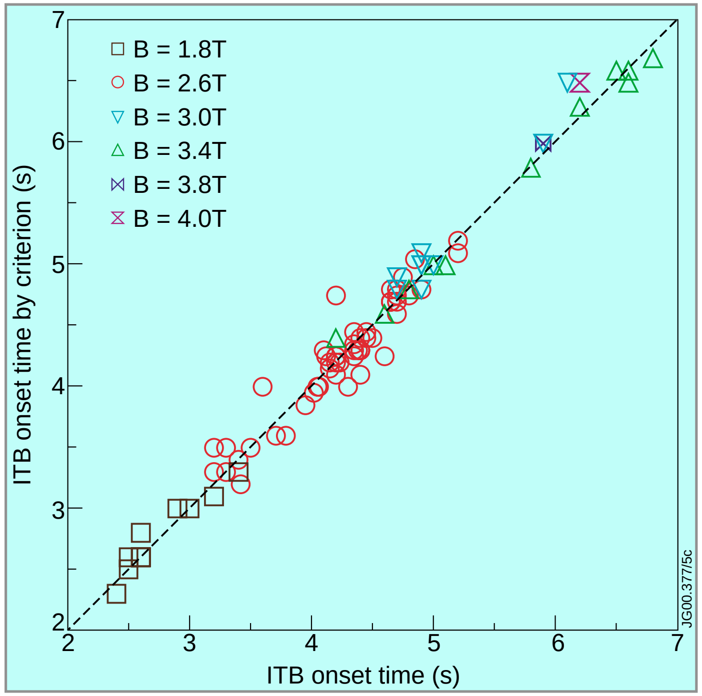
$$\rho^*_T = \rho_s / L_T$$

$$\rho^*_T(R,t) \geq \rho^*_{ITB} \Leftrightarrow \text{ITB at } (R,t)$$

JET Optimised Shear database

$$\rho^*_{ITB}\,(\Lambda_T, s, q, \beta, \nu*, M_\phi, \ldots) \;\approx 1.4 \times 10^{-2}$$

- **116 deuterium pulses**
- **1.8 T ≤ B_ϕ ≤ 4 T**
- **1.6 MA ≤ I_p ≤ 3.6 MA**
- **3.3 ≤ q_{95} ≤ 4.3**
- **2×10^{19} m^{-3} ≤ n_{e0} ≤ 5.5×10^{19} m^{-3}**
- **4.8 MW ≤ P_{NBI} ≤ 18.7 MW**
- **0 MW ≤ P_{ICRH} ≤ 8.7 MW**



G. Tresset et al, Nuclear Fusion 42 (2002) 520

Approximate Model and Singular Value Decomposition

$\mathcal{K}$ = Linear response function ($\mathcal{Y}$ = [current, pressure] ; $\mathcal{P}$ = heating/CD power)

$$\mathcal{Y}(x,t) = \int_0^t dt' \int_0^1 dx' \, \mathcal{K}(x,x',t-t') \, \mathcal{P}(x',t')$$

Laplace transform :

$$\mathcal{Y}(x,s) = \int_0^1 dx' \, \mathcal{K}(x,x',s) \, \mathcal{P}(x',s)$$

Kernel singular value expansion in terms of orthonormal right and left singular functions + System reduction through Truncated SVD (best least square approximation) :

$$\mathcal{K}(x,x',s) = \sum_{i=1}^{\infty} \mathcal{W}_i(x,s) \, \sigma_i(s) \, \overline{\mathcal{V}_i}(x',s)$$

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Set of output trial function basis

Output profiles :

and

Output singular functions :

$$\mathcal{Y}(x,s)=\sum_{j=1}^{N}\mathcal{D}_j(x)\bullet\mathbf{Q}_j(s)+\text{residual}$$

$$\mathcal{W}_k(x,s)=\sum_{j=1}^{N}\mathcal{D}_j(x)\bullet\boldsymbol{\Omega}_{kj}(s)+\text{residual}$$

With 2 profiles (current, pressure) :

$$\mathcal{D}_j(x)=\begin{vmatrix} a_j(x) & 0 \\ 0 & b_j(x) \end{vmatrix}$$

Identification of the operator $\mathcal{K}$

Galerkin's method : residuals spatially orthogonal to each basis function

$$\mathcal{Y}(x,s)=\int_0^1 dx'\,\mathcal{K}(x,x',s)\,\mathcal{P}(x',s)$$

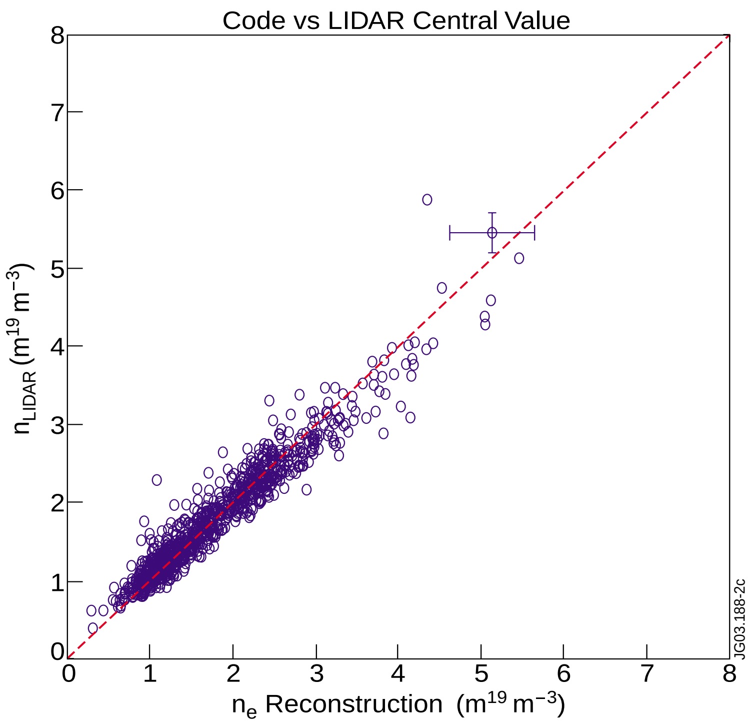
$$\int \text{residual}.\,\mathcal{D}_i(x)\,dx=0$$

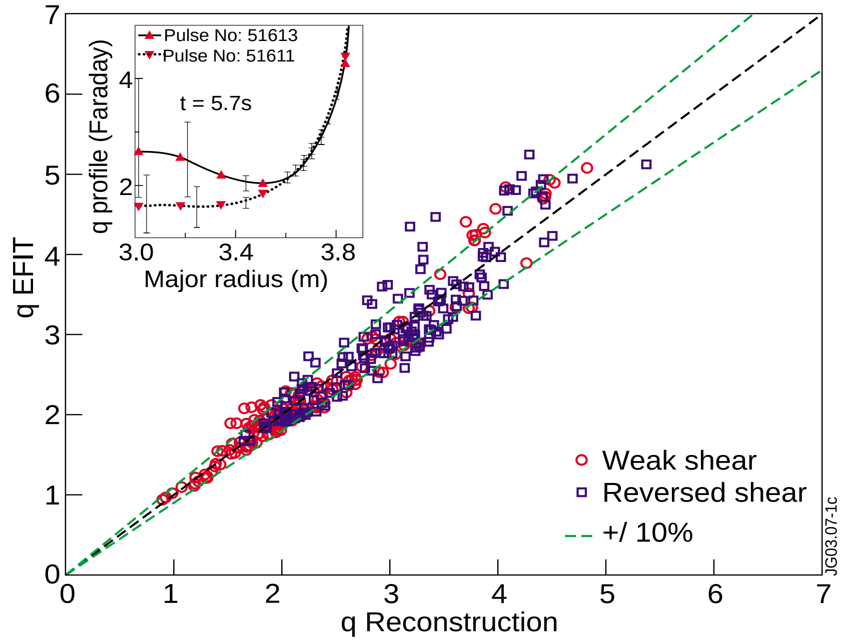
$\Rightarrow$ **Q**(s) = **K**$_{\text{Galerkin}}$(s) . **P**(s)

Real time reconstruction of the safety factor profile (1)

The q-profile reconstruction uses the real-time data from the magnetic measurements and from the interfero-polarimetry, and a parameterization of the magnetic flux surface geometry

$$q(x) = \frac{d\phi}{d\psi} = \frac{c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4}{a_1 x + a_2 x^2 + a_3 x^3}$$





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Trial function basis for q(x) or ι(x)=1/q(x)

If the real-time equilibrium reconstruction uses a particular set of trial functions, then one should take the same set for the controller design.

Otherwise, the family of basis functions must be chosen as to reproduce as closely as possible the family of profiles assumed in the "measurements".

EXAMPLE (with the parameterization used in JET and $c_0 \approx 0$) :

$$q(x) = \frac{d\phi}{d\psi} = \frac{1 + c'_1 x + c'_2 x^2 + c'_3 x^3}{a'_0 + a'_1 x + a'_2 x^2}$$

⇒ 6-parameter family

approximated by $\delta q(x) \text{ or } \iota(x) \approx \sum_1^N Q_i . b_i(x)$ with :

1. A set of N = 6 basis functions $b_i(x)$ can be obtained through differentiation of the rational fraction with respect to the coefficients

2. Alternatively, one can choose $N \leq 6$ cubic splines for $b_i(x)$

What does the controller minimize ?

Output profiles : $\mathcal{Y}(x,s)=\sum_{j=1}^{N}\mathcal{D}_j(x)\bullet\mathbf{Q}_j(s)+\text{residual}$

Setpoint profiles : $\mathcal{Y}_{\text{setpoint}}(x)=\sum_{j=1}^{N}\mathcal{D}_j(x)\bullet\mathbf{Q}_{j,\text{setpoint}}+\text{residual}$

GOAL = minimize $[\mathcal{Y}(s=0)-\mathcal{Y}_{\text{setpoint}}]\bullet[\mathcal{Y}(s=0)-\mathcal{Y}_{\text{setpoint}}]$

Define scalar product to **minimize a least square quadratic form** :

$$\int_0^1 \mu_1(x)\left[q(x)-q_{\text{setpoint}}(x)\right]^2 dx+\int_0^1 \mu_2(x)\left[\rho_T^*(x)-\rho_{T,\text{setpoint}}^*(x)\right]^2 dx$$

Identification of the first singular values and singular functions of $\mathcal{K}$ for the TSVD

$$\mathcal{Y}(x,s) = \int_0^1 dx' \, \mathcal{K}(x,x',s)\, \mathcal{P}(x',s)$$

$$\int_0^1 \text{residual} \, . \, \mathcal{D}_i(x)\, dx = 0$$

EXPERIMENTAL

$\mathbf{Q}(s) = \mathbf{K}_{\text{Galerkin}}(s) \, . \, \mathbf{P}(s)$

The best approximation for σ_k, $\mathcal{W}_k$ and $\mathcal{V}_k$ in the Galerkin sense in the chosen trial function basis $b_i(x)$ is then obtained by performing the SVD of a matrix $\hat{\mathbf{K}}(\mathbf{s})$ related by $\mathbf{K}_{\text{Galerkin}}$ through :

$$B_{i,j} = \int_0^1 \mathbf{b}_i(x) \, . \, \mathbf{b}_j(x)\, dx$$

$\mathbf{B} = \Delta^+ \, . \, \Delta$ (Cholesky decomposition)

$$\hat{\mathbf{K}}(\mathbf{s}) = \boldsymbol{\Delta} \cdot \mathbf{K}_{\mathbf{Galerkin}}(\mathbf{s}) \Longrightarrow \hat{\mathbf{K}}(\mathbf{s}) = \hat{\mathbf{W}}(\mathbf{s}) \cdot \boldsymbol{\Sigma}(\mathbf{s}) \cdot \mathbf{V}^{+}(\mathbf{s}) \Longrightarrow \mathbf{W}(\mathbf{s}) = \boldsymbol{\Delta}^{-1} . \hat{\mathbf{W}}(\mathbf{s})$$

Pseudo-modal control scheme

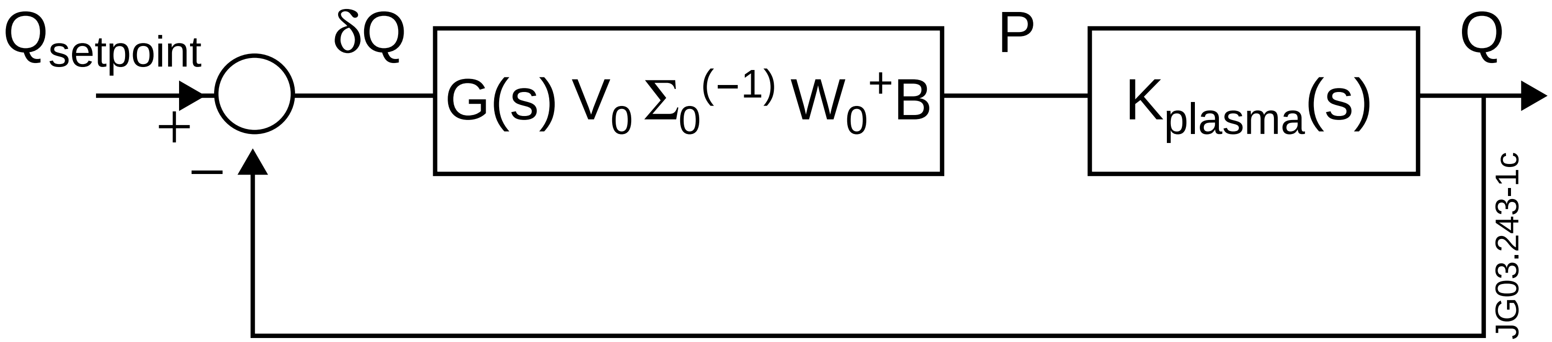
SVD provides decoupled open loop relation between
modal inputs **α**(s) = **V⁺P** and modal outputs **β**(s) = **W⁺BQ**

Truncated diagonal system (≈ 2 or 3 modes) : $\boldsymbol{\beta}(s) = \boldsymbol{\Sigma}(s) \,.\, \boldsymbol{\alpha}(s)$

STEADY STATE DECOUPLING

Use steady state SVD (s=0) to design a Proportional-Integral controller

$$\alpha(s) = G(s).\delta\beta(s) = g_c\,[1+1/(\tau_i.s)]\,.\,\Sigma_0^{-1}.\,\delta\beta(s)$$



Closed-loop transfer function (PI control)

To minimize the difference between the steady state profiles and the reference ones in the least square sense :

$$\mathrm{Min} \int [q(x,s=0) - q_{ref}(x)]^2 \, dx \;\Rightarrow\; \mathrm{Min}\left\{(Q^+ - Q^+_{ref})\, \boldsymbol{\Delta}^+\boldsymbol{\Delta}\, (Q - Q_{ref})\right\}$$

$$\mathbf{Q} = \mathbf{K}_{Galerkin} \,.\, \mathbf{P} \;\Rightarrow\; \text{Solution : } \mathbf{P}_{optimal} = [\mathbf{V_0\, \Sigma_0^{-1}\, W_0^{+}\, B}] \,.\, \mathbf{Q}_{ref}$$

Proposed proportional-integral controller :

$$\mathbf{P}(s) = g_c\, [1+1/(\tau_i.s)] \,.\, [\mathbf{V_0\, \Sigma_0^{-1}\, W_0^{+}\, B}] \,.\, \boldsymbol{\delta Q} = \mathbf{G_c} \,.\, \boldsymbol{\delta Q}$$

- Closed loop transfer function ensures steady state convergence to the least square integral difference with no offset, i. e.

$$\boxed{\mathbf{P}(s=0) = \mathbf{P}_{optimal}}$$

- Choose g_c and τ_i to ensure closed-loop stability [i.e. Im (poles s_k) < 0]

Initial experiments with the lumped-parameter version of the algorithm with 1 actuator q-profile control with LHCD power

The accessible targets are restricted to a **one-parameter family of profiles**

With 5 q-setpoints : no problem if the q-profile tends to "rotate" when varying the power.

With only the internal inductance some features of the q-profile shape could be missed (e.g. reverse or weak shear in the plasma core).

Applying an SVD technique with 5 q-setpoints may not allow to reach any one of the setpoints exactly, but could **minimize the error on the profile shape**.

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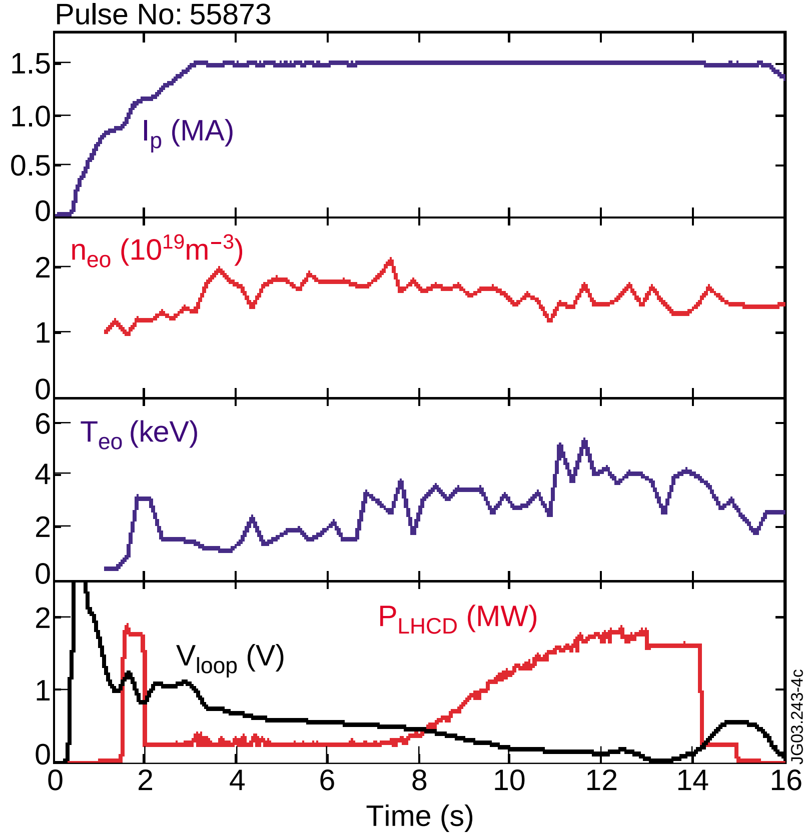
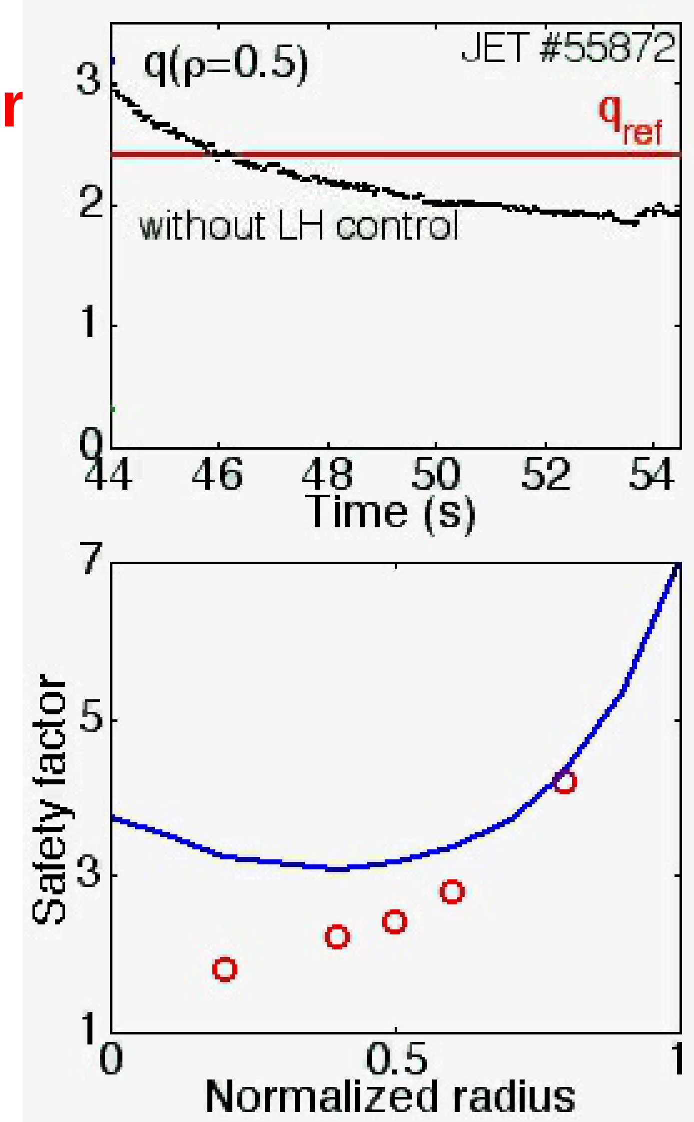
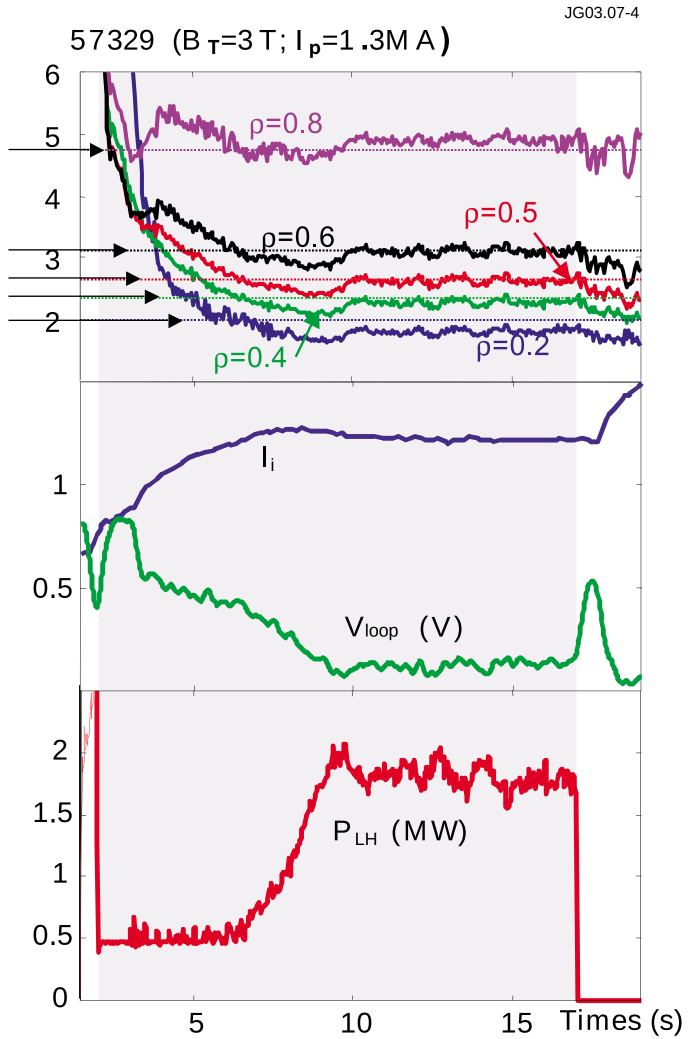
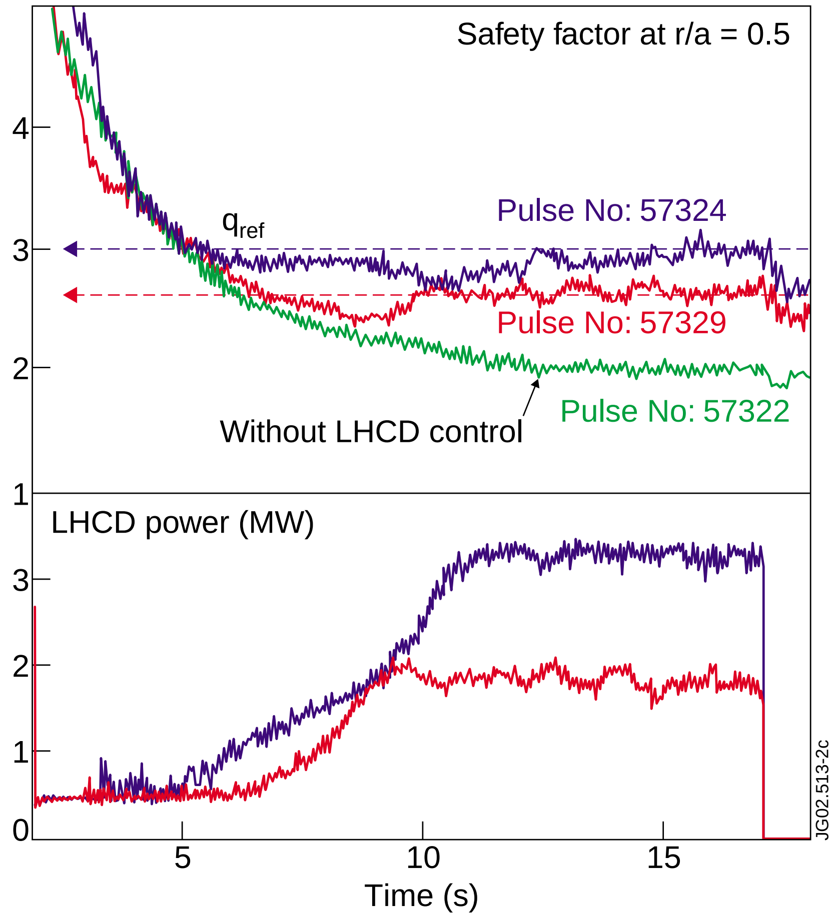
Lumped-parameter version of the algorithm with 1 actuator
q-profile control with LHCD power at 5 radii





5-point q-profile control with LHCD power steady state





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CRONOS integrated modelling code

- Integrated suite of codes, fully modular.
- Solves transport equations (energy, current, density, ...), self-consistently with :
  - Plasma equilibrium (2D equilibrium solver HELENA)
  - Computation of the H/CD/particle sources (ICRH : PION, LHCD : DELPHINE, NBI : SINBAD, ECRH : REMA)
- Coupled to JET, Tore Supra, FTU databases, and ITPA Profile DBs
- Several transport models available : Bohm/gyro-Bohm, Weiland, GLF23, …
- Post-processing : MHD stability (Mishka), diagnostic reconstruction (MSE, polarimetry, …)
- Feedback control available

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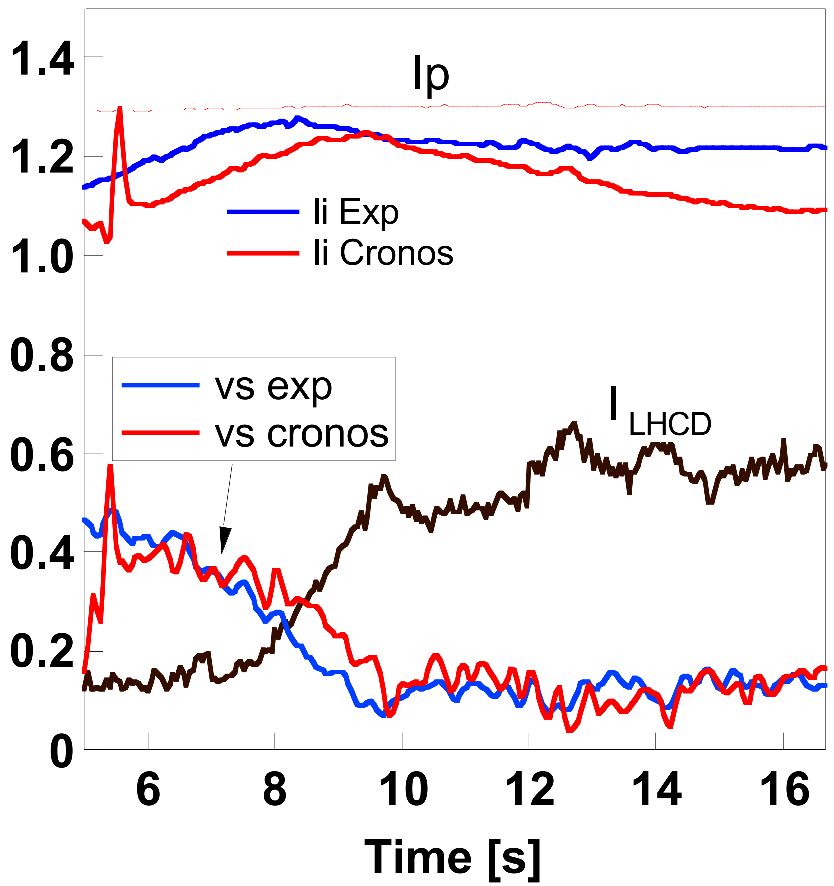
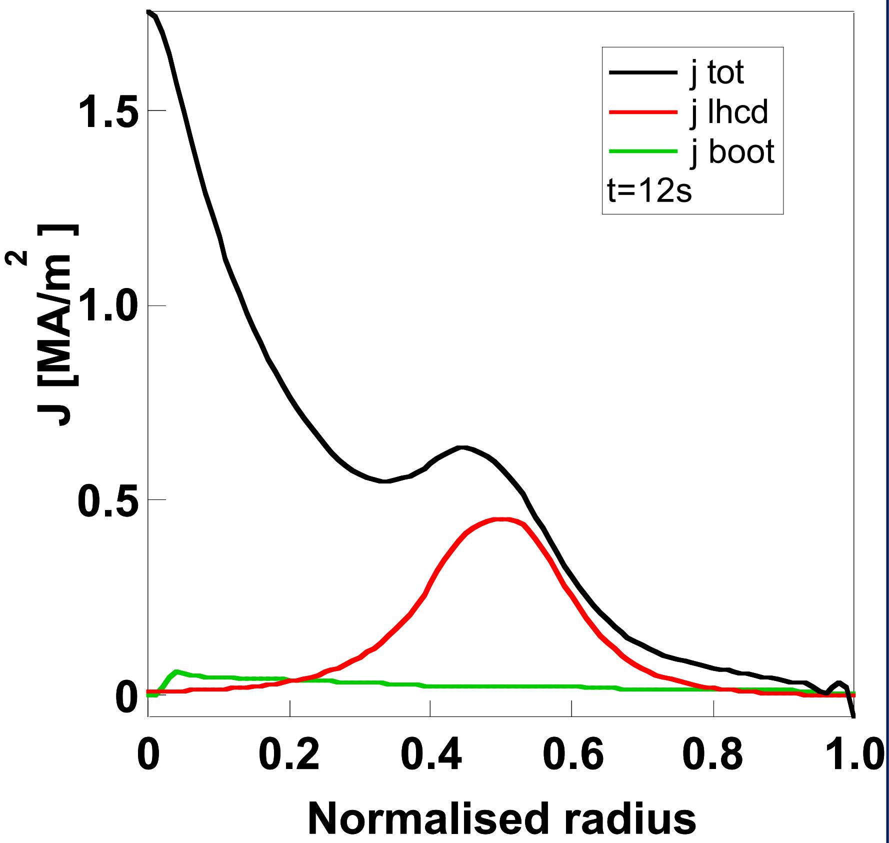
CRONOS Current Diffusion Simulation #57329





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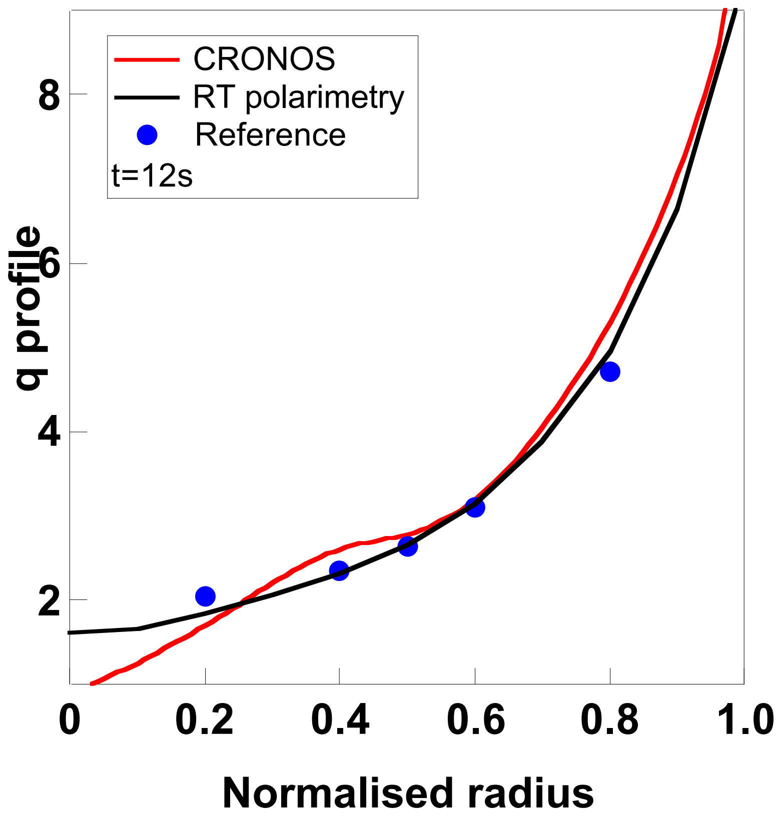
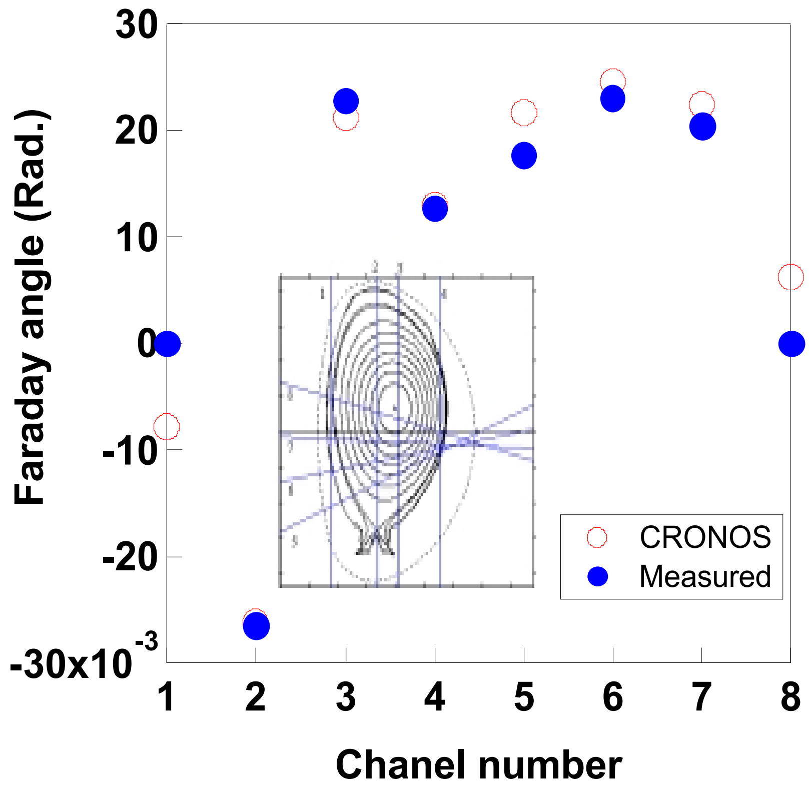
CRONOS Current Diffusion simulation #57329





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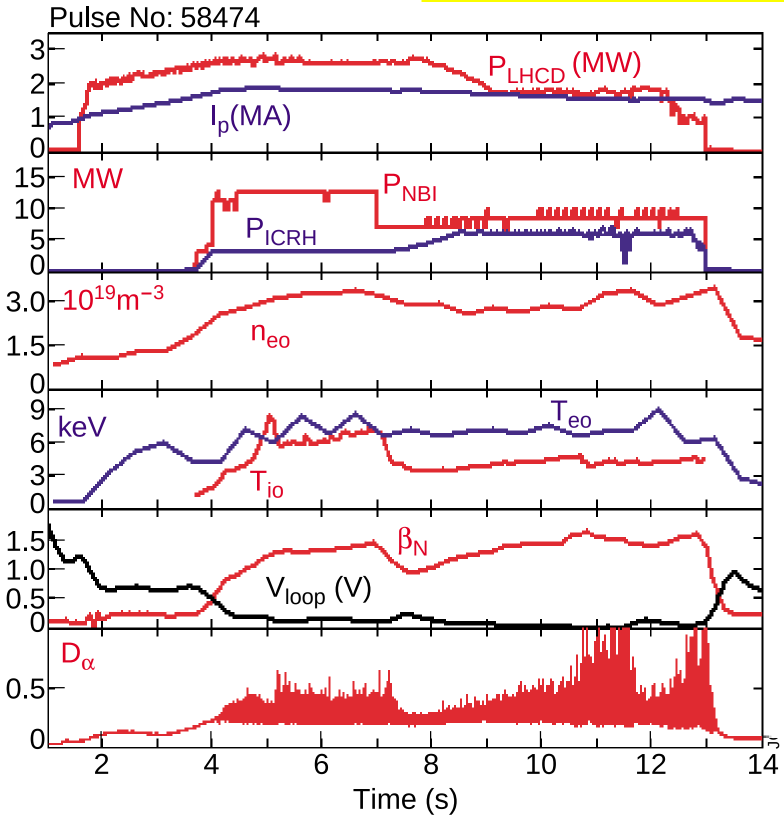
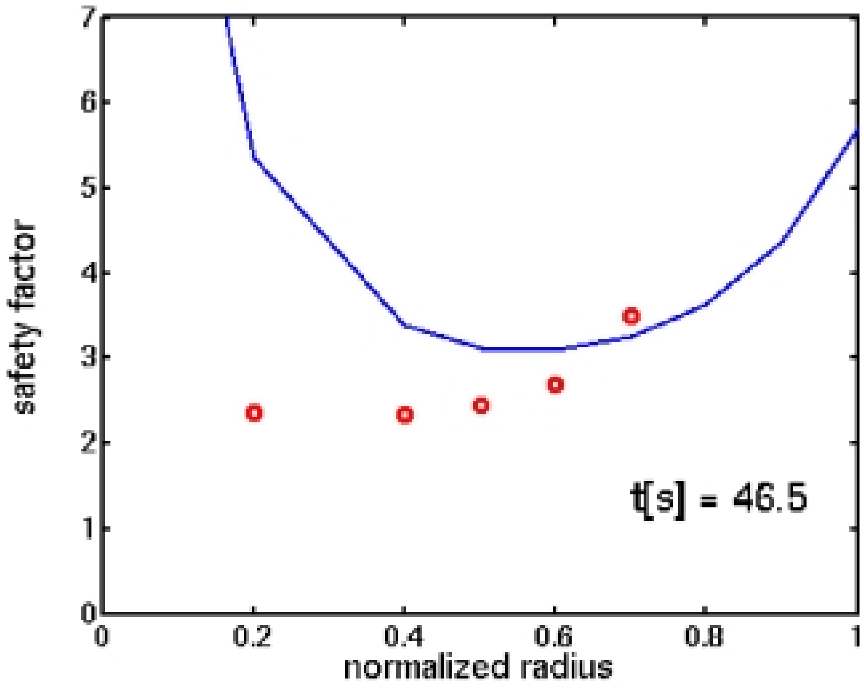
Initial experiments with the lumped-parameter version of the algorithm with 3 actuators
2-mode TSVD for 5-point q-profile control

$$\mathbf{K}_T = \sigma_1 \mathbf{W}_1 . \mathbf{V}_1^{+} + \sigma_2 \mathbf{W}_2 . \mathbf{V}_2^{+}$$





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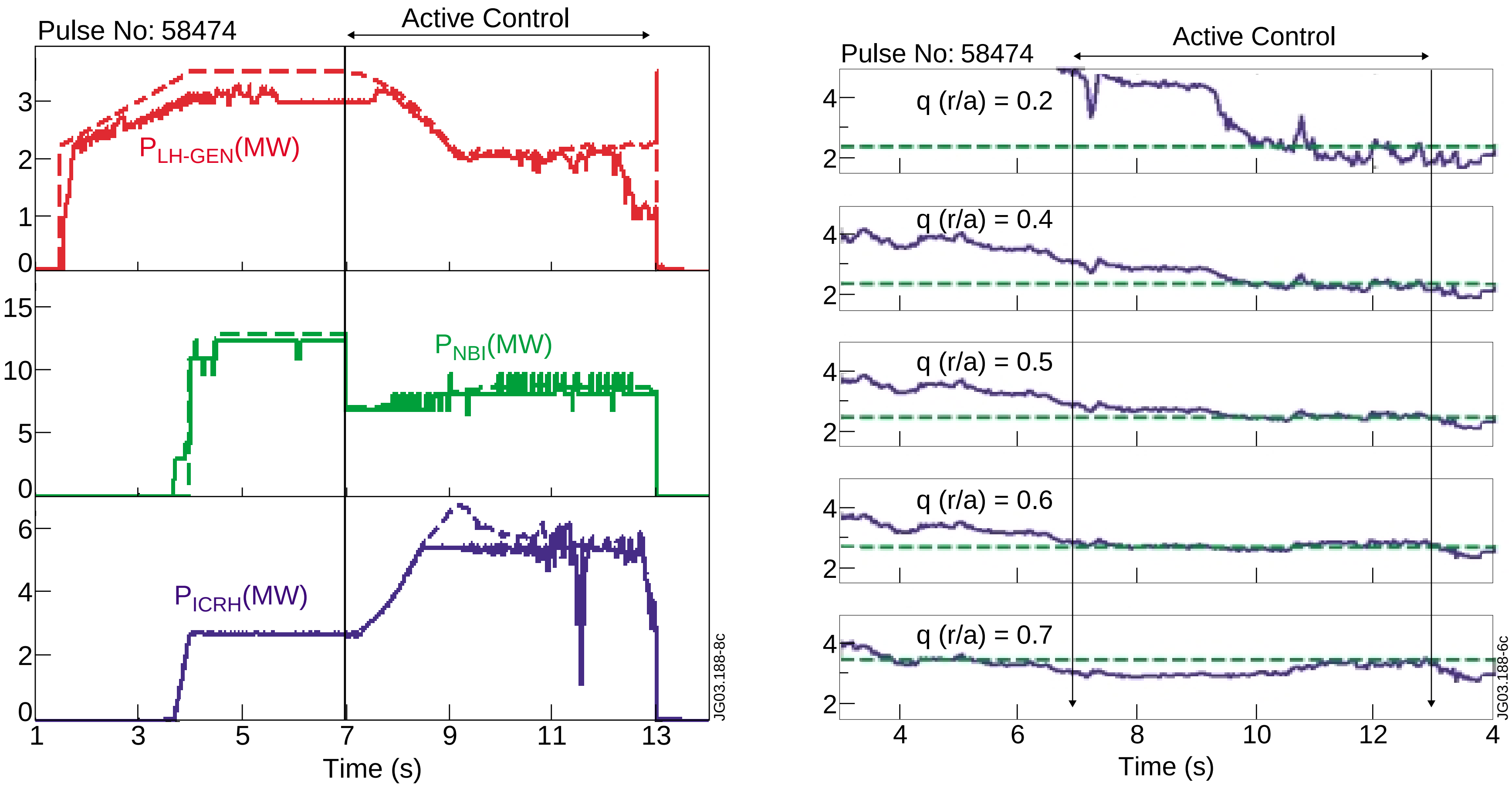

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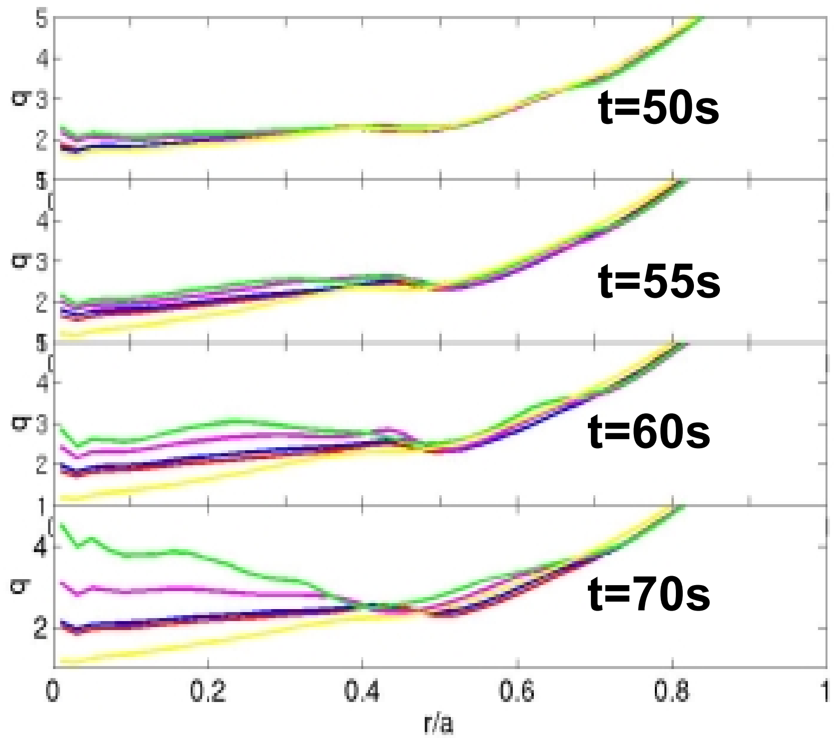
JETTO simulations with the distributed-parameter version of the algorithm with 3 actuators
TSVD for 5-point q-profile control

The Predicted *q*-profile Evolution with RTC

Target q-profiles

- no RTC (ref.)
- flat
- weakly reversed
- reversed
- monotonic



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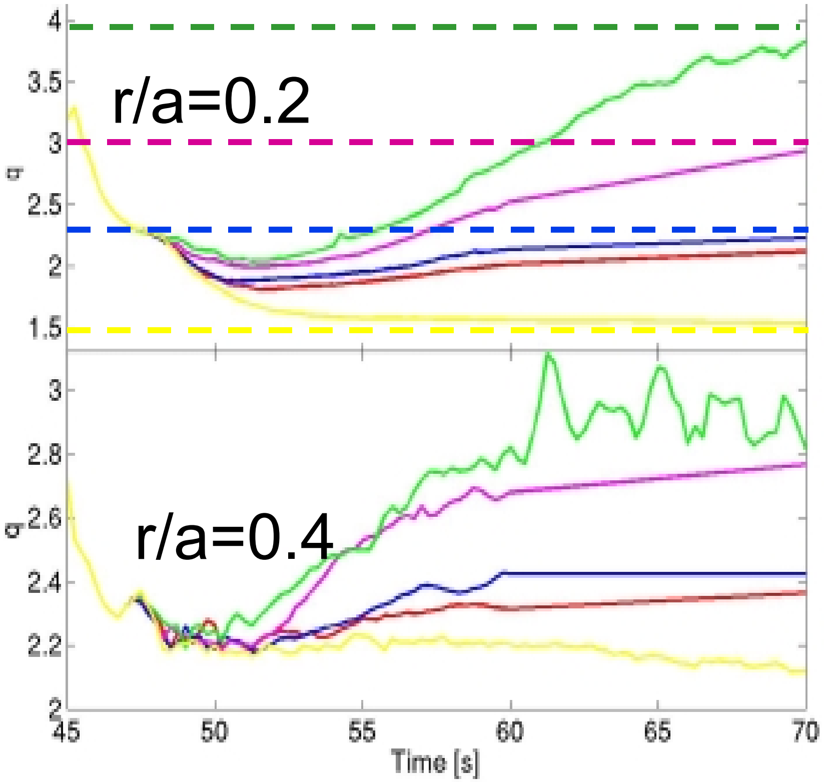

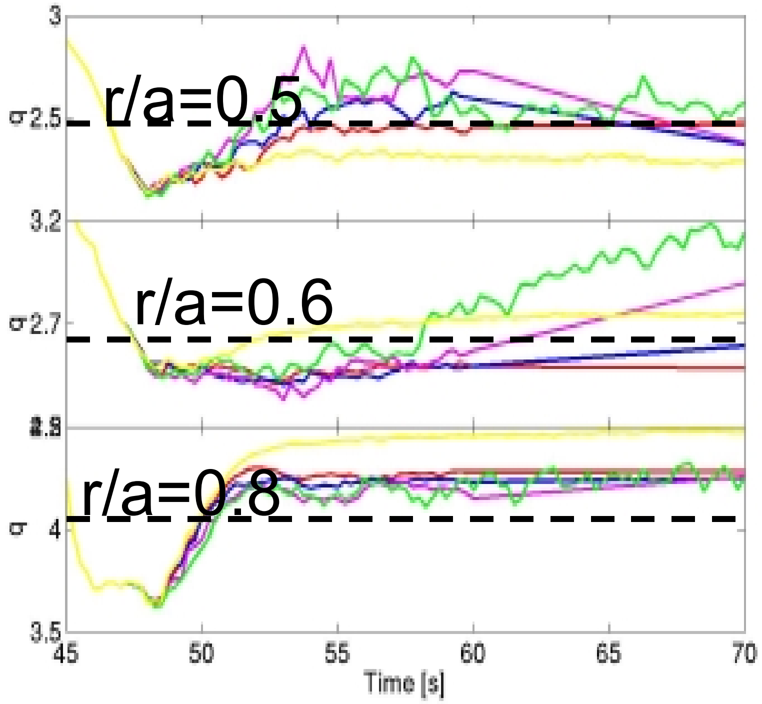

Conclusions and perspectives (1)

1. A **fairly successful control of the safety factor profile** was obtained with the lumped-parameter version of the proposed TSVD algorithm.

2. Preliminary results have just been obtained with the distributed-parameter version **including [q(r) + ρ_T* (r)].**

↓

These results provide an interesting basis and call for a larger integrated modeling and experimental programme on JET, aiming at the sustainement and control of ITB's in fully non-inductive plasmas and **with a large fraction of bootstrap current.**

Conclusions and perspectives (2)

The **potential extrapolability** of the proposed DPS/TSVD technique to **strongly coupled distributed-parameter systems** with a larger number of actuators and input/output parameters and with more flexibility in the deposition profiles, is an attractive feature for an **INTEGRATED BURNING PLASMA CONTROL FOR STEADY STATE ADVANCED REACTOR OPERATION,** including

- **control of the plasma shape,**
- **of the safety factor profile (including plasma current, q_{edge})**
- **of the temperature and density profiles,**
- **but also of the fusion and radiated powers,**
- **and of the primary flux consumption/recharging.**