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Linear Stability of Electrostatic Drift Waves in Helical Plasma~~s~~

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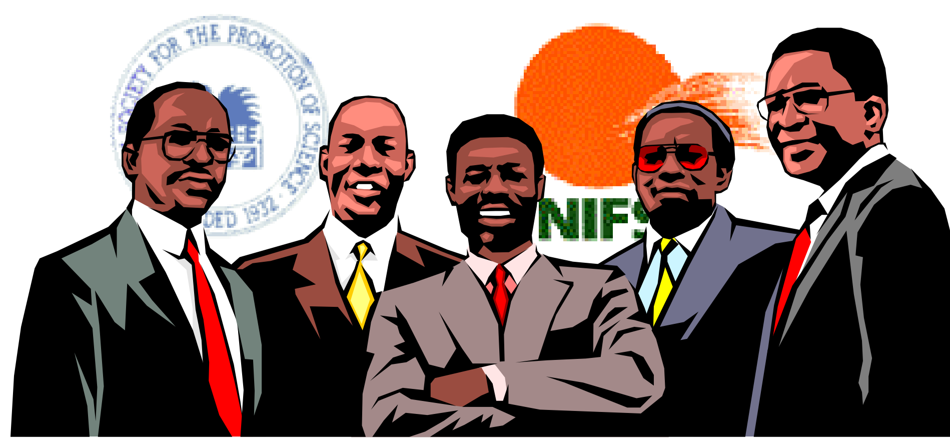


OUTLINE

- Linear stability of **electrostatic** collisionless drift waves in a **helical plasma**
- The stability of different branches (**ITG**,**TEM**,**ETG**)
- **LHD** is considered as a model of helical plasmas

Data analyses for LHD experiments have already been investigated [1,2].
We concentrate on the stability properties of drift waves like,

- What is the local parameter dependence of the modes?
- What is the driving sources in different drift branches?
- are the modes inherent to the helical plasma?
- What is the difference from a tokamak with comparable R_0/a and shear?

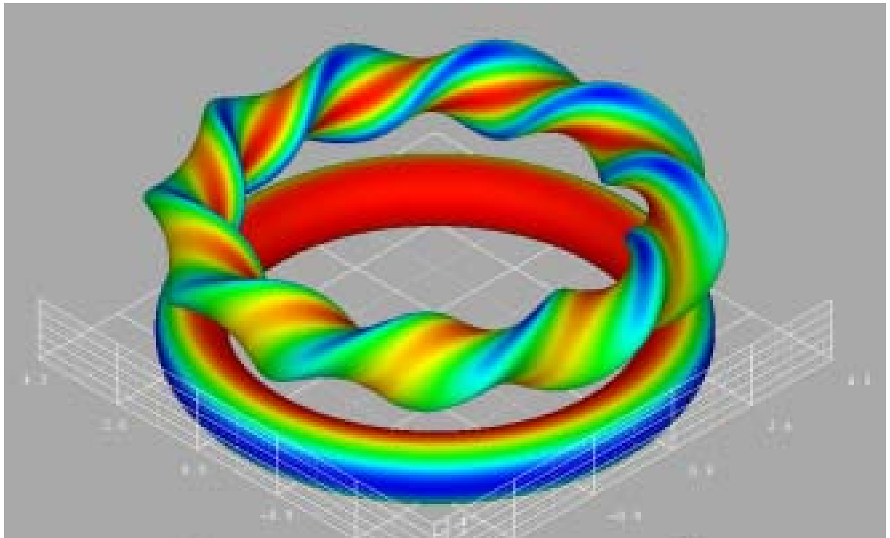
$k_\perp \sim 1/\boldsymbol{\rho}_{thi}$ $\qquad$ $k_\perp \sim \overset{40}{\overset{\gtrsim}{(m_i/m_e)^{1/2}}}/\boldsymbol{\rho}_{thi} = 1/\boldsymbol{\rho}_{the}$

ITG/TEM $\Rightarrow$ $k_\perp$ becomes large (fluctuation becomes small) $\Rightarrow$ ETG

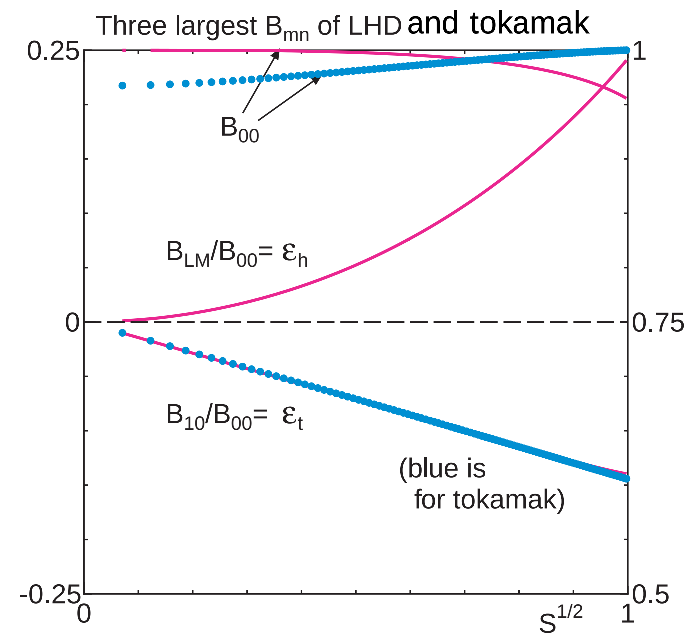
- Realistic 3D MHD configuration obtained by VMEC code[3] is used.
- Formulation is very exact, which uses the ballooning approximation[4].

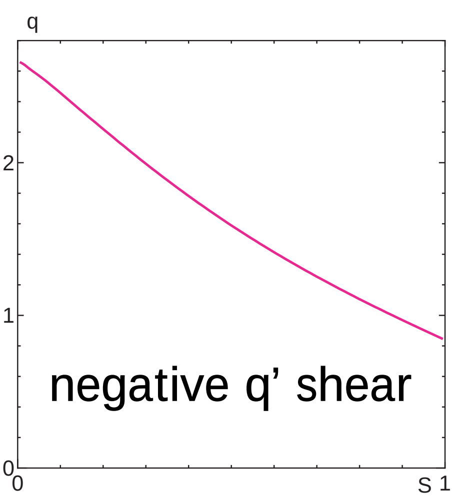
[1]G.Rewoldt, et al., Phys. Plasmas **7**, 4942(2000)
[2]G.Rewoldt, et al., Nucl. Fusion **42**, 1047(2002)
[3]S.P.Hirshman, Phys. Fluids **26**, 3553(1983)
[4]G.Rewoldt, W.M.Tang and M.S.Chance, Phys. Fluids **25**, 480(1982)

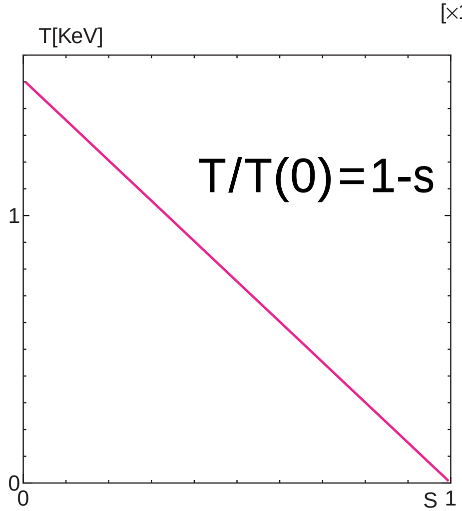
Configurations studied

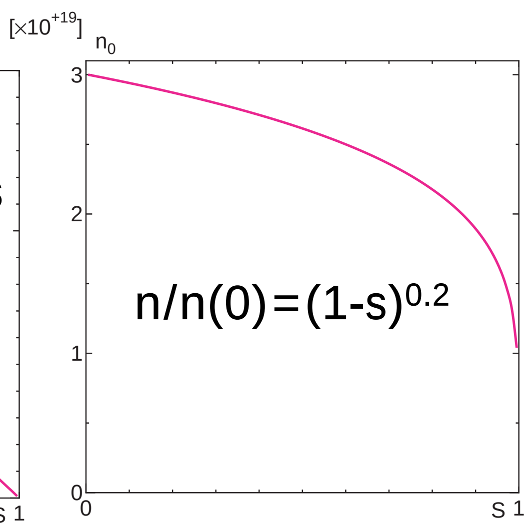


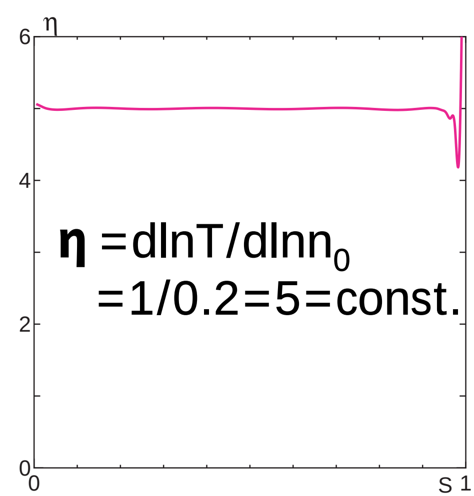
LHD(L=2/M=10: R_0=3.9m/a~0.57m)
and
A comparable aspect ratio **tokamak**











Electrostatic GK eigenmode equation

The perturbed part of the distribution function, δf_j.

The equilibrium distribution is assumed to be Maxwellian, F_{Mj}.

Total distribution function, f_j is decomposed as

$$f_j = F_{Mj} + \delta f_j = F_{Mj} + \frac{-e_j\phi}{T_j} F_{Mj} + h_j,$$

where ϕ is perturbed scalar potential.

Nonadiabatic part of the perturbed distribution function, h_j,

should satisfy the **linear Gyrokinetic equation** [1],

$$\boxed{\left[\omega - \omega_{dj} + i v_{//}\frac{B^\theta}{B}\frac{d}{d\theta}\right] h_{\sigma j}(\theta, E, \Lambda) = \frac{e_j}{T_j}(\omega - \omega_{*j}^T) F_{Mj}(\psi, E) J_0(z_j)\phi(\theta)}$$

The formal solution can be obtained as,

$$h_{\sigma j} = h_{\sigma j}(\theta, E, \Lambda; \omega, \phi).$$

Upon inserting into the Poisson equation, $\nabla^2\phi = -\frac{1}{\varepsilon_0}\sum_j e_j \delta n_j$, we obtain

$$\int d^3v \sum_j e_j \left(\frac{-e_j\phi}{T_j} F_{Mj} + (h_+ + h_-)_j \right) = \varepsilon_0 k_\perp{}^2 \phi.$$

This equation becomes the quasi-neutrality condition when r.h.s is neglected.

To solve, Ritz method is applied, as expanding $\phi = \sum_l (h_l / h)\phi_l$

and integrating with $\frac{T_e}{e_e^2 n_e}\int d\theta h h_{l'}$,

resulting in the matrix eigenvalue problem [2],

$$\sum_{l=0}^{\mathrm{L}} M_{l'l}(\omega)\phi_l = 0, \quad l' = 0, \ldots, L.$$

where

$$\omega_{*j}^T = \omega_{*j}[1 + \eta_j(E - 3/2)],$$

$$\omega_{*j} = \frac{1}{2}\left(\frac{n_k B_0}{\chi'}\right)\frac{1}{L_{nj}}\frac{v_{thj}^2}{\Omega_{0j}},$$

$$\omega_{dj} = -\omega_{*j} L_{nj} E[(2 - \Lambda/h)\kappa_d - (\Lambda/h)\mu_0 p'/B^2],$$

$$F_{Mj} = n_j(\pi v_{thj}^2)^{-3/2}\exp(-E),$$

$E = (v/v_{thj})^2$ and $\Lambda = h(v_\perp/v)^2$ are two dimensionless velocity variables, which are relating to the particle's velocity as

$$v_\perp = v_{thj}\sqrt{E(\Lambda/h)} \text{ and } v_{//} = \sigma v_{thj}\sqrt{E(1-\Lambda/h)},$$

$$d^3v = v_{thj}^3(\pi/2) dE d\Lambda \sqrt{E}\frac{J_0}{h\sqrt{1-\Lambda/h}}, \quad \sigma = \pm 1,$$

$$v_{thj} = \sqrt{2T_j/m_j}, \quad \Omega_{0j} = \frac{e_j B_0}{m_j}, \quad L_{nj} = -\frac{n_j}{n_j{}'}, \quad \eta_j = \frac{d\ln T_j}{d\ln n_j},$$

J_0 is a Bessel function of zeroth order, with argument

$$z_j = k_\perp v_\perp/\Omega_j = k_\perp(v_{thj}/\Omega_{0j})\sqrt{hE\Lambda},$$

$$k_\perp = n_k |(\nabla\varsigma - q\nabla\theta) - q'(\theta - \theta_k)\nabla s|,$$

with n_k being the toroidal mode number,

$h = B_0/B$ with B_0 being the averaged magnetic field at $R = R_0$,

$$\kappa_d = \kappa_n + q'(\theta - \theta_k)\kappa_g$$

with $\kappa_n = \vec{\kappa}\cdot\sqrt{g}\nabla\theta\times\nabla\varsigma$ and $\kappa_g = \vec{\kappa}\cdot\sqrt{g}\nabla s\times\nabla\theta$,

$$h_l = H_l(\sqrt{\delta}\theta)\exp(-(\sqrt{\delta}\theta)^2)/[\sqrt{\pi/\delta}\,2^l l!]$$

with H_l being a Hermite polynomial of order l,

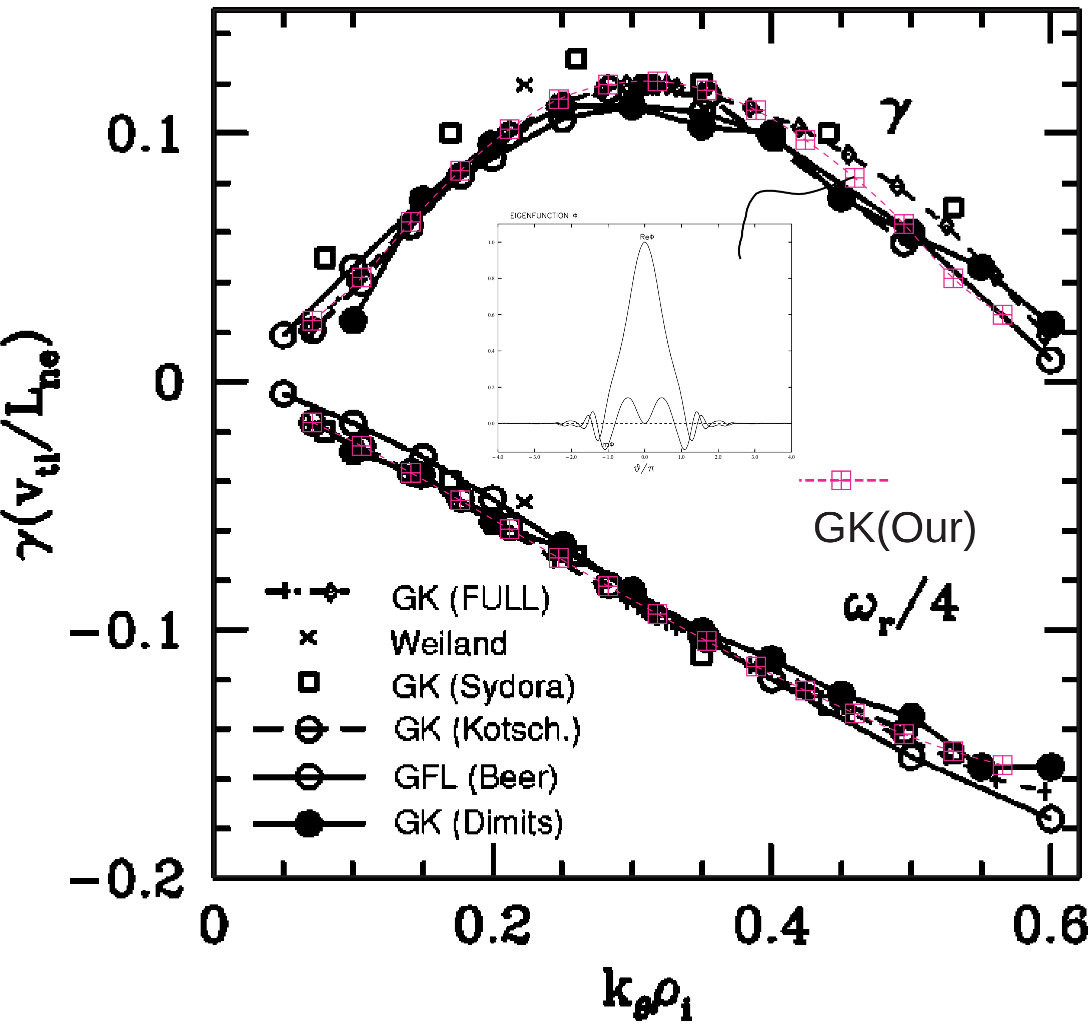
s is normalized toroidal flux, θ and ς are Boozer angle variables,

$d/d\theta = \partial/\partial\theta + q\partial/\partial\varsigma$, and prime denotes the derivative w.r.t s.

[1] J.B.Taylor and R.J.Hastie, Plasma Physics **10**, 479 (1968)

[2] G.Rewoldt, W.M.Tang, and M.S.Chance, Phys. Fluids **25**, 480 (1982)

BENCH MARK



$\varepsilon_t = r/R = 0.18$

$q = 1.4$

$\bar{s} = 0.78, \quad \overline{\alpha} = 0$

$R/L_n = 2.2, \quad R/L_T = 6.9$

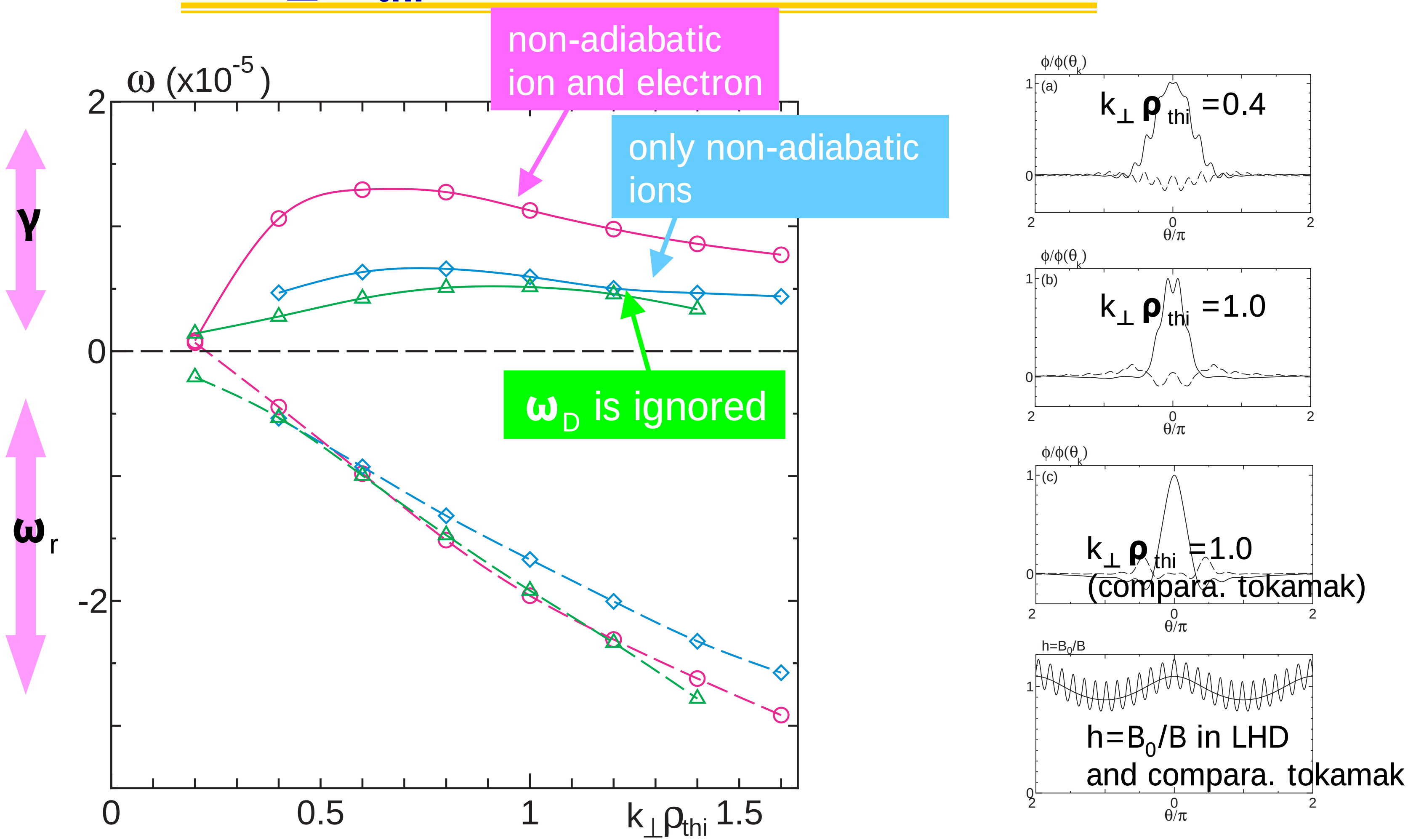
$\eta = L_n/L_T = 3.14$

$m_i/m_e = 3670, \quad T_i/T_e = 1$

s-$\boldsymbol{\alpha}$ equilibrium model is used.
Electrons are assumed to be adiabatic.
Non-adiabatic ions are considered.

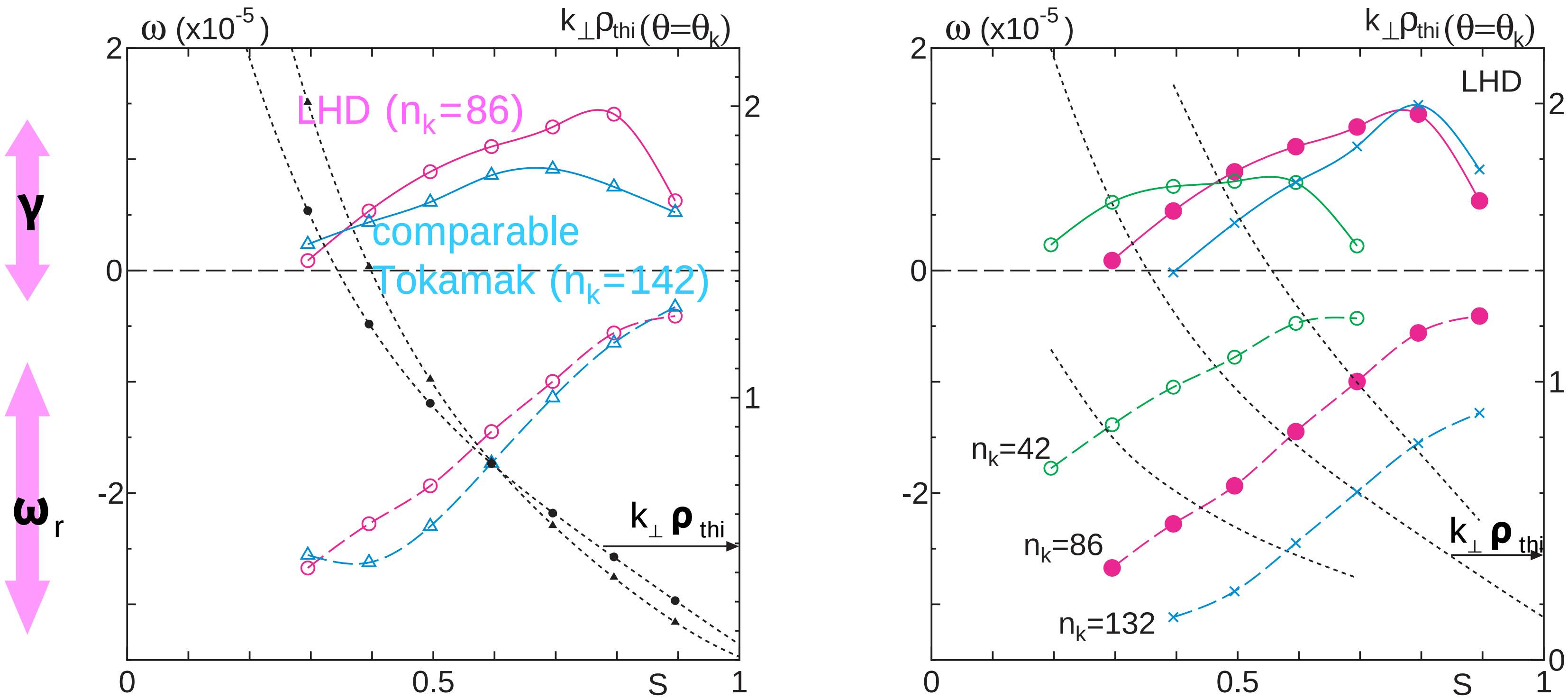
Bench mark in Cyclone base case
(with Dimits et al., Phys. Plasmas 7,969(2000), Fig.1)

$k_\perp \rho_{thi}$ dependence (ITG: in LHD)



- For high $k_\perp \rho_{thi}$, the modes are stabilized (ion FLR effect): Peak is at about $k_\perp \rho_{thi}$~0.6.
- Real frequency is negative ⇒ ITG mode
- The mode can be destabilized without non-adiabatic electrons (ions are essential).
- Non-adiabatic electrons (mainly trapped electrons) further destabilize the modes.
- If ω_D is ignored, the mode can remain unstable due to parallel compressibility (slab-like).
- The mode is not confined in a helical ripple but in a toroidal period.

Radial dependence (ITG: LHD / comparable tokamak)

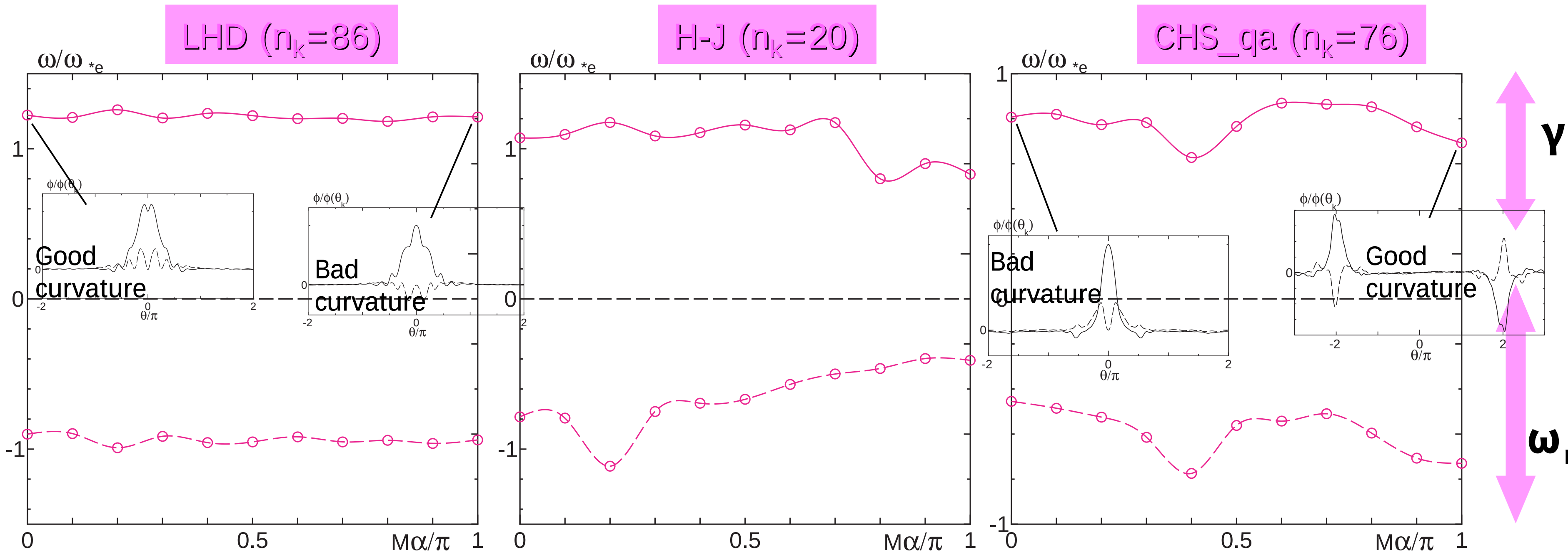


n_k fixed: for LHD and a tokamak

n_k changed: for LHD

- The growth rate peaks at $k_\perp \rho_{thi}$~0.5.
- The core region tends to be stabilized due to high $k_\perp \rho_{thi}$, where T/T(0)=1-s is assumed.
- When changing the toroidal mode number n_k, the peak moves into inside due to the $k_\perp \rho_{thi}$ change.
- Although the local geometrical/magnetic effect and other quantities like q, q' change with s, these effects seem to be weak, and the dependence is almost determined by value of $k_\perp \rho_{thi}$.
- Thus the tendency is common for tokamak and LHD, and the frequency is not so different.

α dependence (ITG: s=0.7)



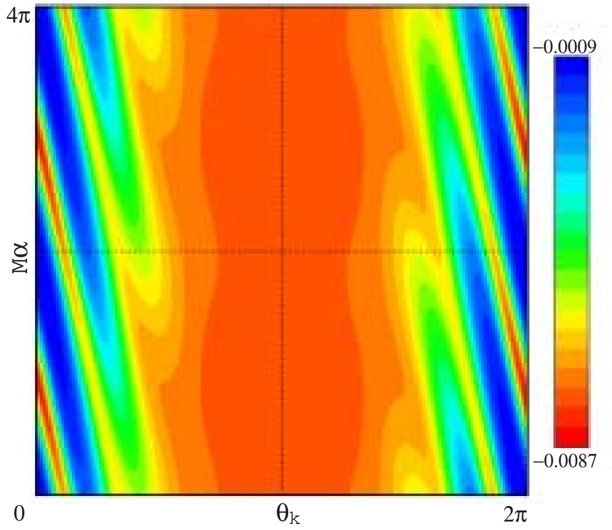
In all cases, the **α dependence is weak**.

In the LHD, the mode is extended over several helical ripples (one helical ripple ~ **π**/M= **π**/10).
In the CHS_qa (and Heliotron J) the mode is almost localized into a helical ripple ~ **π**/2 or 4.

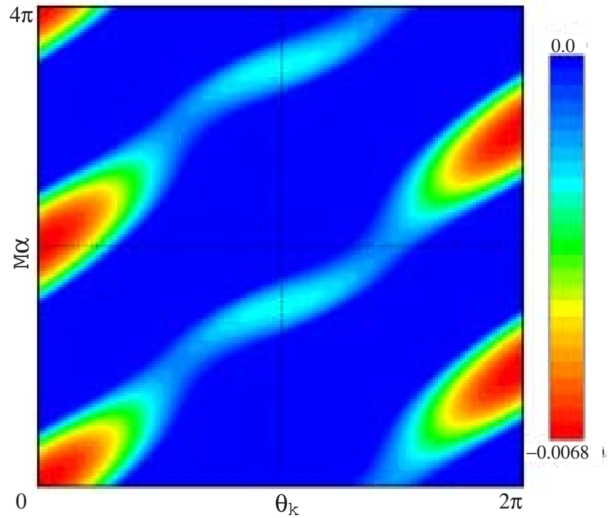
In the LHD, the mode is insensitive to the change of **α** due to their extended nature.
In the CHS_qa and Heliotron J, the frequency is also unchanged with **α**,
but in this case, due to the localization into a bad helical ripple.

α dependence (MHD: n=∞)

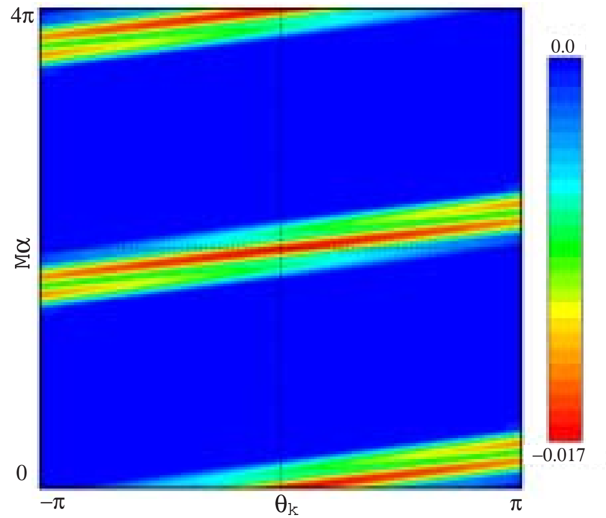
LHD(s=0.55)



H-J(s=0.55)

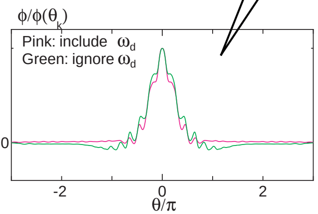


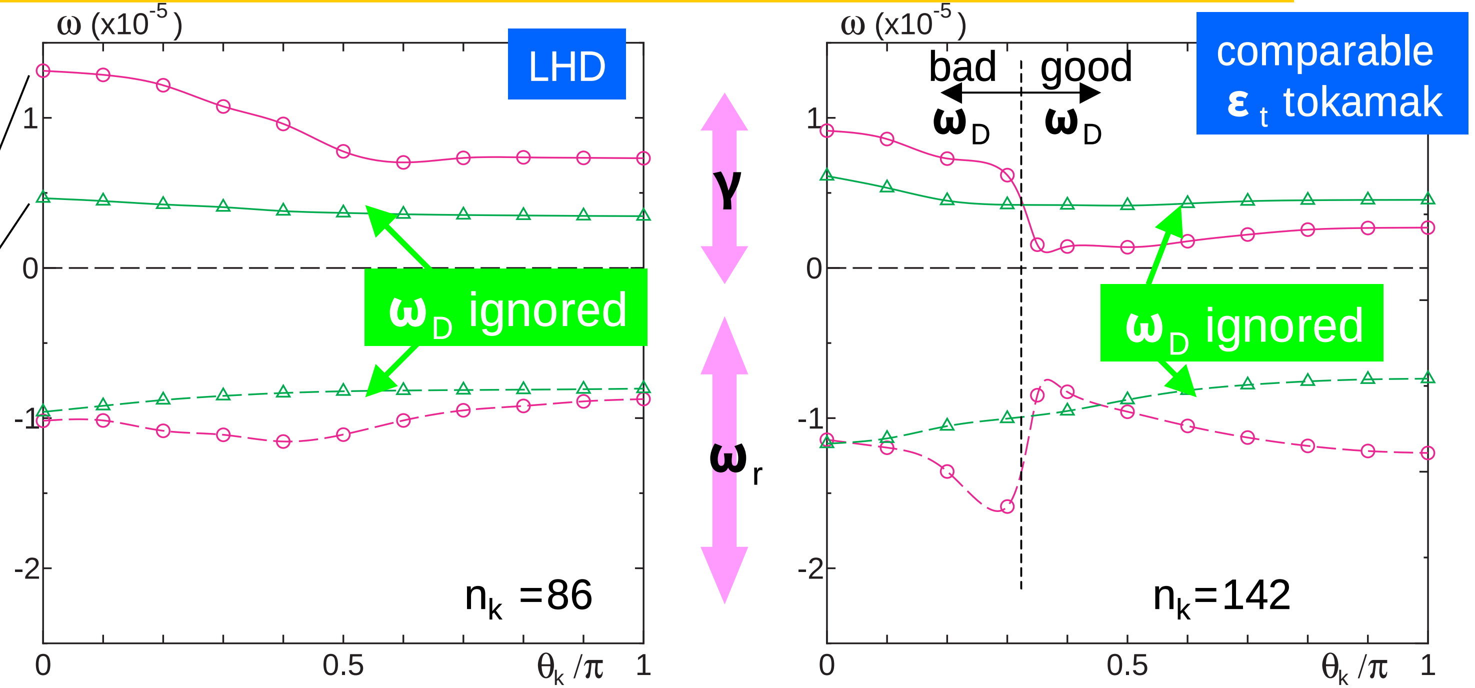
CHS_qa(s=0.5)



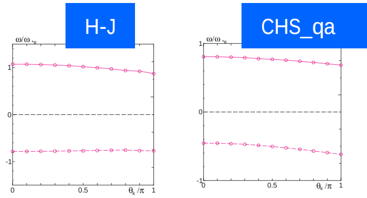
Magneto Hydro Dynamics modes show the strong **α** dependence in helical systems

θ_k dependence (ITG: LHD / a tokamak)



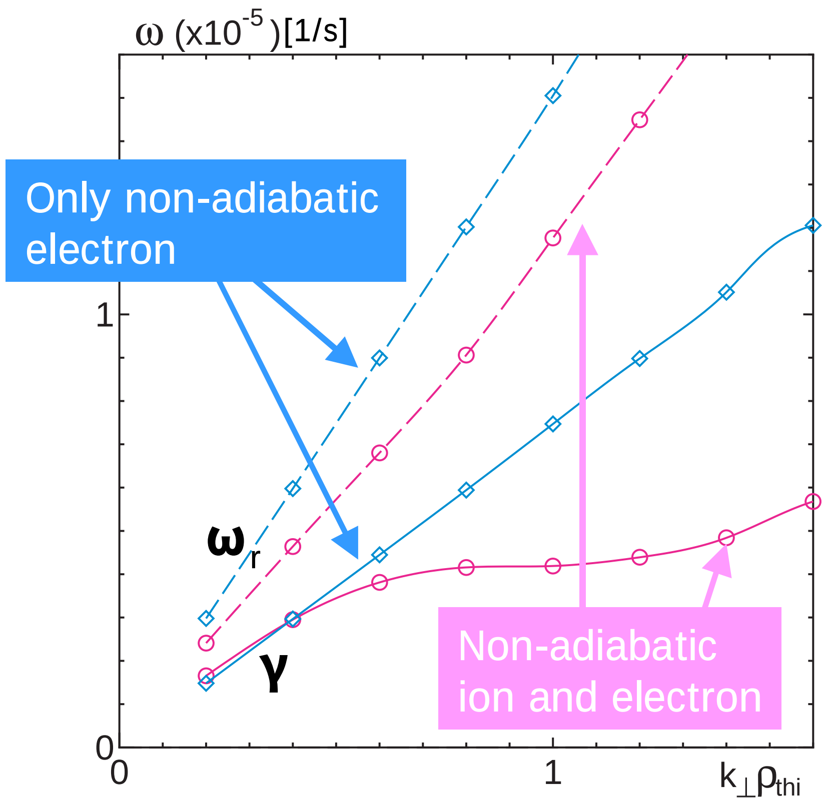


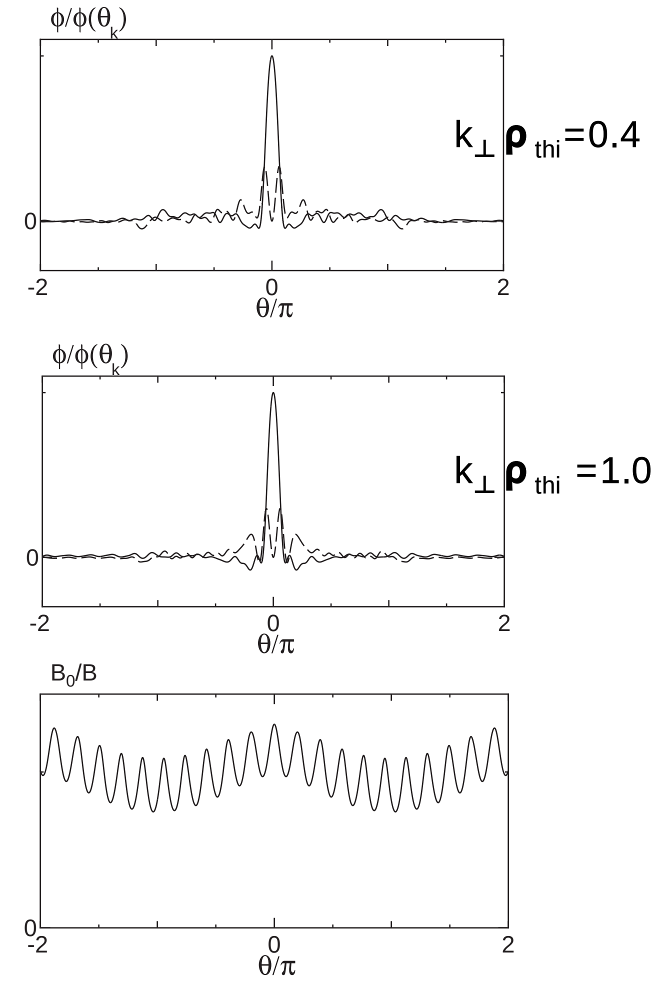
- The θ_k dependence is weaker than that in MHD ballooning(modes are not stabilized completely).
- When ω_D is ignored, the θ_k dependence becomes more weak (slab-like modes).
- γ is comparable in LHD and tokamak, and ω_r is larger in LHD than in tokamak.
- ω_D is destabilizing at θ_k~0 for both LHD and tokamak.
  However it becomes stabilizing in comparable tokamak as θ_k increases.



>Slab-like, parallel compressibility makes the modes unstable for all θ_k (weak θ_k dependence).
>The bad curvature (perpendicular compressibility) further destabilizes the modes through ω_D.
>In tokamak, good curvature in the inboard side is stabilizing.
>In LHD, locally bad curvature region extends to inboard side, and ω_D is always destabilizing.

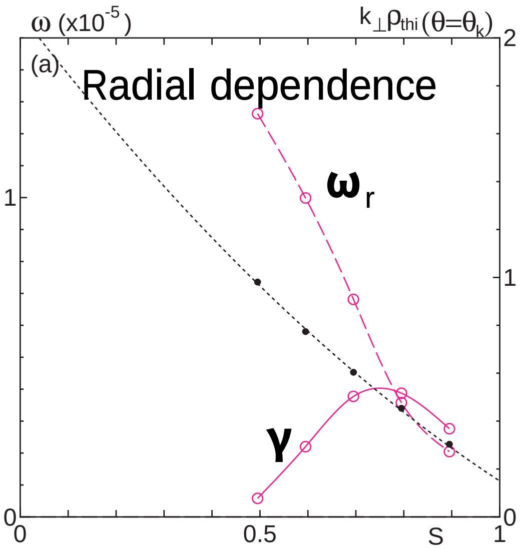
TEM case (in LHD: s=0.7)

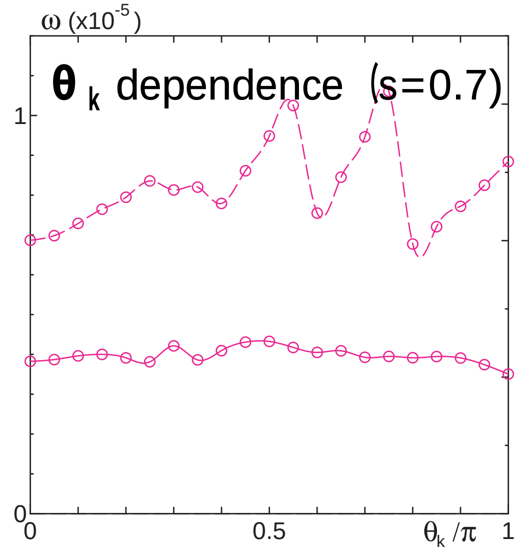


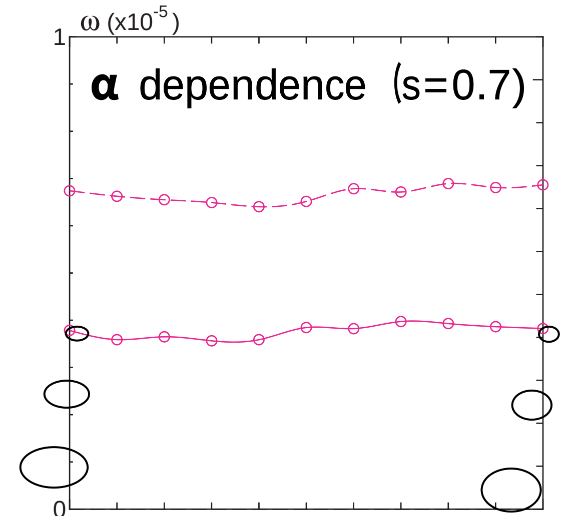


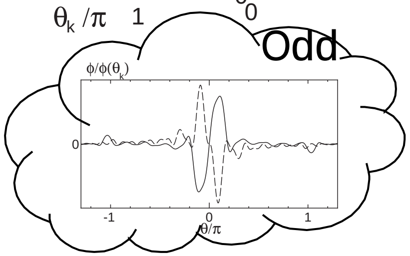
- Real frequency is positive.
- Adiabatic-ion case is unstable ⇒ electrons are essential ⇒TEMs
- Increase of $k_\perp \boldsymbol{\rho}_{thi}$ **cannot** stabilize the modes ⇒ $\boldsymbol{\rho}_{the} \ll \boldsymbol{\rho}_{thi}$ (ion FLR does not work).
- The mode is strongly localized in a helical ripple.
- If $\boldsymbol{\omega}_D$ is ignored, the modes are stabilized ($\boldsymbol{\omega}_D$ is essential, which is different from ITG).
- Inclusion of non-adiabatic ions is stabilizing, which is contrary to the ITG case.
  (The reason is now under consideration)

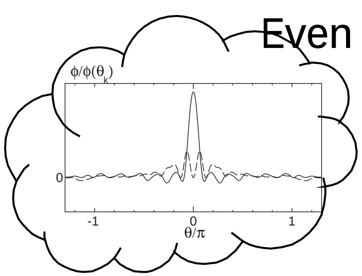
Local parameter dependence (TEM in LHD)











Toroidal mode number n_k=86: fixed

i) Plasma core region is stabilized,
which is not due to high $k_\perp \rho_{thi}$ value but due to small ε_t, ε_h.

ii) Local curvature is more bad toward edge (ε_t, ε_h is increasing function in magnitude).
The competition between bad curvature and small $k_\perp \rho_{thi}$ would determine the growth rate.

iii) θ_k and α dependence is found to be weak,
because the mode can find another helical ripple to localize when θ_k or α is changed.

ETG case

Gyrokinetic equation

$$\left[\omega - \omega_{dj} + i v_{//}\frac{B^{\theta}}{B}\frac{d}{d\theta}\right] h_{\sigma j} = \frac{e_j}{T_j}(\omega - \omega_{*j}^{T}) F_{Mj} J_0 \phi,$$

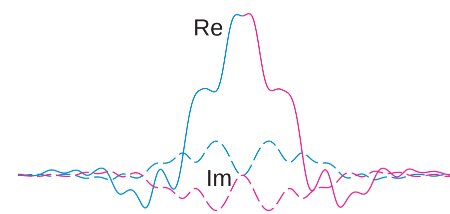
Poisson equation

$$\int d^3v \sum_j e_j \left(\frac{-e_j \phi}{T_j} F_{Mj} + (h_+ + h_-)_j \right) = \varepsilon_0 k_\perp{}^2 \phi$$

Assume that the ITG/ETG is destabilized only by non-adiabatic ions/electrons.
Then, two types of instabilities can be treated merely as the **scale transformation**.
(isomorphism)

Normalize the eigenvalue for ITG/ETG as, $\hat{\omega}_{jTG} = \omega / (v_{thj} / L_{nj}), \quad j = i \text{ or } e$
then two types of instabilities can satisfy the gyrokinetic-Poisson equations, as long as

$$\hat{\omega}_{ETG} = \hat{\omega}_{ITG} \quad \text{and} \quad \begin{array}{l} \mathrm{Re}[\phi_{ETG}] = \mathrm{Re}[\phi_{ITG}] \text{ and } \mathrm{Im}[\phi_{ETG}] = -\mathrm{Im}[\phi_{ITG}], \\ \mathrm{Re}[h_{ETG}] = -\mathrm{Re}[h_{ITG}] \text{ and } \mathrm{Im}[h_{ETG}] = \mathrm{Im}[h_{ITG}] \end{array}$$



Trapped ions | Circulating ions

Trapped electrons | Circulating electrons

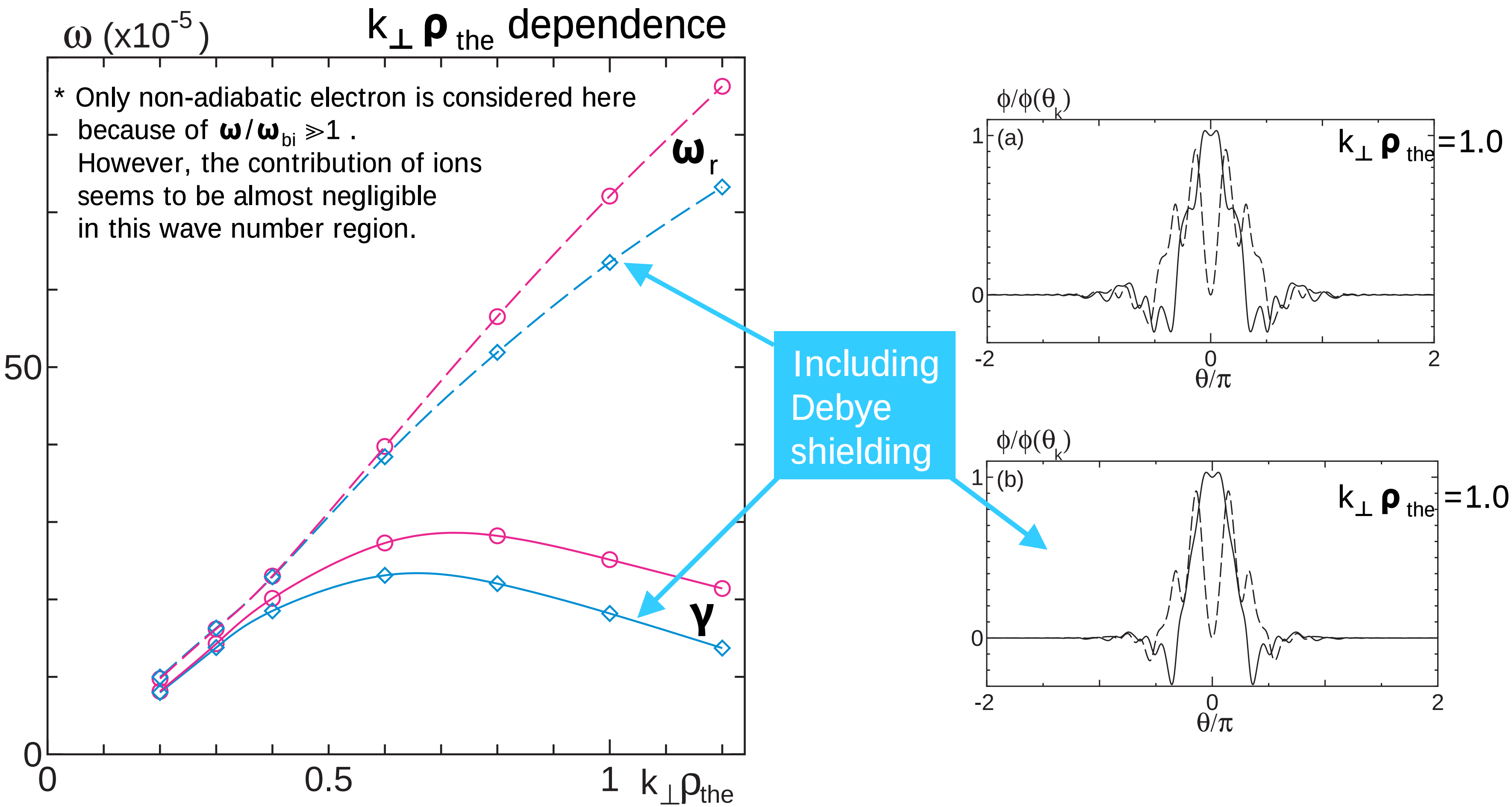
Debye shielding effects

fine, rapid phenomena

The minor changes come from

i) Trapped electron contribution to the ITG
ii) Debye shielding effect on the ETG

$k_\perp \rho_{the}$ dependence (ETG: in LHD)



- Real frequency is positive ⇒ ETG mode.
- For high $k_\perp \rho_{the}$, the modes are stabilized (electron FLR + Debye shielding).
- Debye shielding is stabilizing at higher $k_\perp \rho_{the}$, while it is negligible for $k_\perp \rho_{the} \lesssim 0.3$.

Summary

Linear stability of collisionless electrostatic drift modes (ITG, TEM, ETG) in a helical plasma, LHD is investigated.

- ITG: occurs at $k_\perp \rho_{thi} \sim 1$. The ITG can be destabilized only by the slab-like mechanism without curvature, and the (helical) curvature is side effect on the instabilities. Thus, the frequencies are not so different between the LHD and a tokamak, when the comparable aspect ratio and the same (negative) magnetic shear are assumed.
  The mode structure is extended along the field line (in a toroidal period $\sim 2\pi$).
  Then the local parameter (α, θ_k) dependence is found to be weak.
  Plasma core region tends to be stable for the ITG due to the ion FLR effects.

- TEM: occurs at $k_\perp \rho_{thi} \sim 1$. Contrary to the ITG, the TEM is strongly localized in a helical ripple$\sim 2\pi/M$.
  This is considered that helically trapped electrons in LHD is main source of the destabilization.
  The increase of $k_\perp \rho_{thi}$ linearly destabilizes the modes unlike the case of ITG.
  This is reasonable since the ion FLR should not work for electron's modes.
  The local parameter (α, θ_k) dependence is found to be weak, due to their localized nature.
  In LHD the TEM can be found even in the negative q' while they are stabilized in tokamak.
  Plasma core tends to be stabilized as in the case of the ITG,
  but in this case, due to the decrease of local curvature ($\varepsilon_t, \varepsilon_h$).

- ETG: occurs at $k_\perp \rho_{the} \sim 1$. This is about $(m_i/m_e)^{1/2}$ times higher mode number modes than the ITG.
  In this case, $k_\perp \lambda_D \sim 1$, and the Debye shielding effect becomes important.
  Oppositely the shielding effect is weak for $k_\perp \rho_{the} \ll 1$,
  and quasi-neutrarity is good approximation for the wave region of ITG/TEM.
  The other physical properties of ETG should be almost the same as the ITG,
  because ETG can be considered as the nearly scale transformation from the ITG.