

1 Equations in r coordinates

$$0 = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \phi + \frac{1}{\epsilon_0} (-en_e + Z_i en_i + Z_b en_b + Z_b e g n_b^{\text{rp}}) \quad (1.1)$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} \dot{A}_\theta = \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} r A_\theta + \mu_0 (-en_e u_{e\theta} + Z_i en_i u_{i\theta} + Z_b en_b u_{b\theta}) \quad (1.2)$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} \dot{A}_\phi = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} A_\phi + \mu_0 (-en_e u_{e\phi} + Z_i en_i u_{i\phi} + Z_b en_b u_{b\phi}) \quad (1.3)$$

$$\frac{\partial}{\partial t} A_\phi = \dot{A}_\phi \quad (1.4)$$

$$\frac{\partial}{\partial t} A_\theta = \dot{A}_\theta \quad (1.5)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e = & -\frac{1}{r} \frac{\partial}{\partial r} r n_e u_{er} + \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) - \nu_L (n_e - n_{\text{ediv}}) \\ & + (1 - f_{\text{CX}}) \frac{P_b}{E_b} \end{aligned} \quad (1.6)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e u_{er} = & -\frac{1}{r} \frac{\partial}{\partial r} r u_{er} n_e u_{er} + \frac{1}{r} u_{e\theta} n_e u_{e\theta} - \frac{\partial}{\partial r} \frac{n_e T_e}{m_e} \\ & - \frac{e}{m_e} n_e E_r - \frac{e}{m_e} B_\phi n_e u_{e\theta} + \frac{e}{m_e} B_\theta n_e u_{e\phi} \end{aligned} \quad (1.7)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e u_{e\theta} = & -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 u_{er} n_e u_{e\theta} + \frac{1}{r^2} \frac{\partial}{\partial r} r^3 n_e \mu_e \frac{\partial}{\partial r} \frac{u_{e\theta}}{r} - \frac{e}{m_e} n_e E_\theta + \frac{e}{m_e} B_\phi n_e u_{er} \\ & - \nu_{\text{NCe}} n_e u_{e\theta} - \nu_{\text{ei1}} n_e (u_{e\theta} - u_{i\theta}) - \nu_{\text{ei2}} n_e (u_{e\phi} - u_{i\phi}) \\ & - \frac{m_b}{m_e} \nu_{\text{be1}} n_b (u_{e\theta} - u_{b\theta}) - \frac{m_b}{m_e} \nu_{\text{be2}} n_b (u_{e\phi} - u_{b\phi}) \\ & - \frac{e^2 B_\phi^2 D_e}{m_e T_e} n_e \left[(u_{e\theta} - u_{i\theta}) - \frac{B_\theta}{B_\phi} (u_{e\phi} - u_{i\phi}) + \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e r + \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial r} T_e + \frac{E_r}{B_\phi} \right] \\ & - 2\nu_L n_e u_{e\theta} - \nu_{0e} n_e u_{e\theta} + \mu_{e2} \frac{B_\phi}{B_\theta^2} \frac{1}{m_e e} \frac{\partial}{\partial r} T_e \end{aligned} \quad (1.8)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e u_{e\phi} = & -\frac{1}{r} \frac{\partial}{\partial r} r u_{er} n_e u_{e\phi} + \frac{1}{r} \frac{\partial}{\partial r} r n_e \mu_e \frac{\partial}{\partial r} u_{e\phi} - \frac{e}{m_e} n_e E_\phi - \frac{e}{m_e} B_\theta n_e u_{er} \\ & - \nu_{\text{ei3}} n_e (u_{e\phi} - u_{i\phi}) - \nu_{\text{ei2}} n_e (u_{e\theta} - u_{i\theta}) - \frac{m_b}{m_e} \nu_{\text{be2}} n_b (u_{e\theta} - u_{b\theta}) - \frac{m_b}{m_e} \nu_{\text{be3}} n_b (u_{e\phi} - u_{b\phi}) \\ & + \frac{e^2 B_\phi^2 D_e}{m_e T_e} \frac{B_\theta}{B_\phi} n_e \left[(u_{e\theta} - u_{i\theta}) - \frac{B_\theta}{B_\phi} (u_{e\phi} - u_{i\phi}) + \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e r + \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial r} T_e + \frac{E_r}{B_\phi} \right] \end{aligned}$$

$$- 2\nu_{\text{L}}n_{\text{e}}u_{\text{e}\phi} - \nu_{0\text{e}}n_{\text{e}}u_{\text{e}\phi} \quad (1.9)$$

$$\begin{aligned} \frac{\partial}{\partial t} \frac{3}{2} n_{\text{e}} T_{\text{e}} = & -\frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{5}{2} u_{\text{er}} n_{\text{e}} T_{\text{e}} - n_{\text{e}} \chi_{\text{e}} \frac{\partial}{\partial r} T_{\text{e}} \right) + u_{\text{er}} \frac{\partial}{\partial r} n_{\text{e}} T_{\text{e}} - e E_{\theta} n_{\text{e}} u_{\text{e}\theta} - e E_{\phi} n_{\text{e}} u_{\text{e}\phi} \\ & - \frac{3}{2} \nu_{\text{Tei}} n_{\text{e}} (T_{\text{e}} - T_{\text{i}}) - \nu_{\text{L}} T_{\text{e}} (n_{\text{e}} - n_{\text{ediv}}) - \frac{3}{2} \nu_{\text{LT}_{\text{e}}} n_{\text{e}} (T_{\text{e}} - T_{\text{ediv}}) \\ & - E_{\text{H}} \nu_{\text{ionize}} \frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) + f_{\text{ei}} P_{\text{b}} + P_{\alpha\text{e}} + P_{\text{RFe}} - P_{\text{Br}} \end{aligned} \quad (1.10)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_{\text{i}} = & -\frac{1}{r} \frac{\partial}{\partial r} r n_{\text{i}} u_{\text{ir}} + \frac{\nu_{\text{ionize}}}{Z_{\text{i}}} \frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) - \frac{\nu_{\text{L}}}{Z_{\text{i}}} (n_{\text{e}} - n_{\text{ediv}}) \\ & + \nu_{\text{b}} n_{\text{b}} + \nu_{\text{b}} g n_{\text{b}}^{\text{rp}} - f_{\text{CX}} \frac{P_{\text{b}}}{E_{\text{b}}} + \nu_{\text{bL}} n_{\text{b}} + S_{\text{LC}} - \nu_{\text{OL}} n_{\text{i}} \end{aligned} \quad (1.11)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_{\text{i}} u_{\text{ir}} = & -\frac{1}{r} \frac{\partial}{\partial r} r u_{\text{ir}} n_{\text{i}} u_{\text{ir}} + \frac{1}{r} u_{\text{i}\theta} n_{\text{i}} u_{\text{i}\theta} - \frac{\partial}{\partial r} \frac{n_{\text{i}} T_{\text{i}}}{m_{\text{i}}} \\ & + Z_{\text{i}} \frac{e}{m_{\text{i}}} n_{\text{i}} E_r + Z_{\text{i}} \frac{e}{m_{\text{i}}} B_{\phi} n_{\text{i}} u_{\text{i}\theta} - Z_{\text{i}} \frac{e}{m_{\text{i}}} B_{\theta} n_{\text{i}} u_{\text{i}\phi} \end{aligned} \quad (1.12)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_{\text{i}} u_{\text{i}\theta} = & -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 u_{\text{ir}} n_{\text{i}} u_{\text{i}\theta} + \frac{1}{r^2} \frac{\partial}{\partial r} r^3 n_{\text{i}} \mu_{\text{i}} \frac{\partial}{\partial r} \frac{u_{\text{i}\theta}}{r} + Z_{\text{i}} \frac{e}{m_{\text{i}}} n_{\text{i}} E_{\theta} - Z_{\text{i}} \frac{e}{m_{\text{i}}} B_{\phi} n_{\text{i}} u_{\text{ir}} \\ & - \nu_{\text{NCi}} n_{\text{i}} u_{\text{i}\theta} - \frac{m_{\text{e}}}{m_{\text{i}}} \nu_{\text{ei1}} n_{\text{e}} (u_{\text{i}\theta} - u_{\text{e}\theta}) - \frac{m_{\text{e}}}{m_{\text{i}}} \nu_{\text{ei2}} n_{\text{e}} (u_{\text{i}\phi} - u_{\text{e}\phi}) - \frac{m_{\text{b}}}{m_{\text{i}}} \nu_{\text{bi}} n_{\text{b}} (u_{\text{i}\theta} - u_{\text{b}\theta}) \\ & + \frac{e^2 B_{\phi}^2 D_{\text{e}}}{m_{\text{i}} T_{\text{e}}} n_{\text{e}} \left[(u_{\text{e}\theta} - u_{\text{i}\theta}) - \frac{B_{\theta}}{B_{\phi}} (u_{\text{e}\phi} - u_{\text{i}\phi}) + \frac{B}{B_{\phi}} \left\langle \frac{\omega}{m} \right\rangle_{\text{e}} r + \frac{1 - \alpha_{\text{e}}}{e B_{\phi}} \frac{\partial}{\partial r} T_{\text{e}} + \frac{E_r}{B_{\phi}} \right] \\ & - 2\nu_{\text{L}} n_{\text{i}} u_{\text{i}\theta} - \nu_{0\text{i}} n_{\text{i}} u_{\text{i}\theta} - \nu_{\text{CX}} n_{\text{i}} u_{\text{i}\theta} + S_{\text{LC}\theta} - \nu_{\text{OL}} n_{\text{i}} u_{\text{i}\theta} - \mu_{\text{i}2} \frac{B_{\phi}}{B_{\theta}^2} \frac{1}{m_{\text{i}} Z_{\text{i}} e} \frac{\partial}{\partial r} T_{\text{i}} + T_{\theta,\text{i}} \end{aligned} \quad (1.13)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_{\text{i}} u_{\text{i}\phi} = & -\frac{1}{r} \frac{\partial}{\partial r} r u_{\text{ir}} n_{\text{i}} u_{\text{i}\phi} + \frac{1}{r} \frac{\partial}{\partial r} r n_{\text{i}} \mu_{\text{i}} \frac{\partial}{\partial r} u_{\text{i}\phi} + Z_{\text{i}} \frac{e}{m_{\text{i}}} n_{\text{i}} E_{\phi} + Z_{\text{i}} \frac{e}{m_{\text{i}}} B_{\theta} n_{\text{i}} u_{\text{ir}} \\ & - \frac{m_{\text{e}}}{m_{\text{i}}} \nu_{\text{ei3}} n_{\text{e}} (u_{\text{i}\phi} - u_{\text{e}\phi}) - \frac{m_{\text{e}}}{m_{\text{i}}} \nu_{\text{ei2}} n_{\text{e}} (u_{\text{i}\theta} - u_{\text{e}\theta}) - \frac{m_{\text{b}}}{m_{\text{i}}} \nu_{\text{bi}} n_{\text{b}} (u_{\text{i}\phi} - u_{\text{b}\phi}) \\ & - \frac{e^2 B_{\phi}^2 D_{\text{e}}}{m_{\text{i}} T_{\text{e}}} \frac{B_{\theta}}{B_{\phi}} n_{\text{e}} \left[(u_{\text{e}\theta} - u_{\text{i}\theta}) - \frac{B_{\theta}}{B_{\phi}} (u_{\text{e}\phi} - u_{\text{i}\phi}) + \frac{B}{B_{\phi}} \left\langle \frac{\omega}{m} \right\rangle_{\text{e}} r + \frac{1 - \alpha_{\text{e}}}{e B_{\phi}} \frac{\partial}{\partial r} T_{\text{e}} + \frac{E_r}{B_{\phi}} \right] \\ & - 2\nu_{\text{L}} n_{\text{i}} u_{\text{i}\phi} - \nu_{0\text{i}} n_{\text{i}} u_{\text{i}\phi} - \nu_{\text{CX}} n_{\text{i}} u_{\text{i}\phi} + S_{\text{LC}\phi} - \nu_{\text{OL}} n_{\text{i}} u_{\text{i}\phi} + T_{\phi,\text{i}} \end{aligned} \quad (1.14)$$

$$\begin{aligned} \frac{\partial}{\partial t} \frac{3}{2} n_{\text{i}} T_{\text{i}} = & -\frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{5}{2} u_{\text{ir}} n_{\text{i}} T_{\text{i}} - n_{\text{i}} \chi_{\text{i}} \frac{\partial}{\partial r} T_{\text{i}} \right) + u_{\text{ir}} \frac{\partial}{\partial r} n_{\text{i}} T_{\text{i}} + Z_{\text{i}} e E_{\theta} n_{\text{i}} u_{\text{i}\theta} + Z_{\text{i}} e E_{\phi} n_{\text{i}} u_{\text{i}\phi} \\ & - \frac{3}{2} \nu_{\text{Tei}} n_{\text{e}} (T_{\text{i}} - T_{\text{e}}) + m_{\text{b}} v_{\text{b}} \frac{B_{\theta} u_{\text{i}\theta} + B_{\phi} u_{\text{i}\phi}}{B} \frac{P_{\text{b}\parallel}}{E_{\text{b}}} - \frac{\nu_{\text{L}}}{Z_{\text{i}}} T_{\text{i}} (n_{\text{e}} - n_{\text{ediv}}) \\ & - \frac{3}{2} \nu_{\text{LT}_{\text{i}}} n_{\text{i}} (T_{\text{i}} - T_{\text{idiv}}) + \frac{3}{2} \frac{\nu_{\text{ionize}}}{Z_{\text{i}}} \frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}} (n_{01} T_{01} + n_{02} T_{02} + n_{03} T_{03}) \end{aligned}$$

$$-\frac{3}{2}\nu_{\text{CX}}n_{\text{i}}(T_{\text{i}}-T_{01})+(1-f_{\text{ei}})P_{\text{b}}+P_{\text{ai}}+P_{\text{RFi}} \quad (1.15)$$

$$\frac{\partial}{\partial t}n_{\text{b}}=\frac{P_{\text{b}}}{E_{\text{b}}}-g\frac{P_{\text{b}\perp}}{E_{\text{b}}}-\nu_{\text{b}}n_{\text{b}}-\nu_{\text{bL}}n_{\text{b}}-\nu_{\text{eff}}g(\sqrt{\delta}n_{\text{b}}-n_{\text{b}}^{\text{rp}})+\frac{1}{r}\frac{\partial}{\partial r}rf_{\text{t}}D_{\text{b}}\frac{\partial}{\partial r}n_{\text{b}} \quad (1.16)$$

$$\begin{aligned} \frac{\partial}{\partial t}n_{\text{b}}u_{\text{b}\theta} &= Z_{\text{b}}\frac{e}{m_{\text{b}}}n_{\text{b}}E_{\theta}-\nu_{\text{be1}}n_{\text{b}}(u_{\text{b}\theta}-u_{\text{e}\theta})-\nu_{\text{be2}}n_{\text{b}}(u_{\text{b}\phi}-u_{\text{e}\phi})-\nu_{\text{bi}}n_{\text{b}}(u_{\text{b}\theta}-u_{\text{i}\theta}) \\ &\quad -\nu_{0\text{b}}n_{\text{b}}u_{\text{b}\theta}-\nu_{\text{CX}}n_{\text{b}}u_{\text{b}\theta}+v_{\text{b}\theta}\frac{P_{\text{b}\parallel}}{E_{\text{b}}}-\nu_{\text{bL}}n_{\text{b}}u_{\text{b}\theta}-\nu_{\text{eff}}g\sqrt{\delta}n_{\text{b}}u_{\text{b}\theta} \\ &\quad +\frac{1}{r}\frac{\partial}{\partial r}ru_{\text{b}\theta}f_{\text{t}}D_{\text{b}}\frac{\partial}{\partial r}n_{\text{b}} \end{aligned} \quad (1.17)$$

$$\begin{aligned} \frac{\partial}{\partial t}n_{\text{b}}u_{\text{b}\phi} &= Z_{\text{b}}\frac{e}{m_{\text{b}}}n_{\text{b}}E_{\phi}-\nu_{\text{be3}}n_{\text{b}}(u_{\text{b}\phi}-u_{\text{e}\phi})-\nu_{\text{be2}}n_{\text{b}}(u_{\text{b}\theta}-u_{\text{e}\theta})-\nu_{\text{bi}}n_{\text{b}}(u_{\text{b}\phi}-u_{\text{i}\phi}) \\ &\quad -\nu_{0\text{b}}n_{\text{b}}u_{\text{b}\phi}-\nu_{\text{CX}}n_{\text{b}}u_{\text{b}\phi}+v_{\text{b}\phi}\frac{P_{\text{b}\parallel}}{E_{\text{b}}}-\nu_{\text{bL}}n_{\text{b}}u_{\text{b}\phi}-\nu_{\text{eff}}g\sqrt{\delta}n_{\text{b}}u_{\text{b}\phi} \\ &\quad +\frac{1}{r}\frac{\partial}{\partial r}ru_{\text{b}\phi}f_{\text{t}}D_{\text{b}}\frac{\partial}{\partial r}n_{\text{b}} \end{aligned} \quad (1.18)$$

$$\begin{aligned} \frac{\partial}{\partial t}n_{01} &= \frac{1}{r}\frac{\partial}{\partial r}rD_{01}\frac{\partial}{\partial r}n_{01}-\frac{1}{Z_{\text{i}}}\nu_{\text{ionize}}\frac{n_{\text{e}}}{n_{01}+n_{02}+n_{03}}n_{01}-\nu_{\text{CX}}\frac{n_{\text{i}}}{n_{01}+n_{02}}n_{01} \\ &\quad +\gamma_0\frac{\nu_{\text{L}}}{Z_{\text{i}}}(n_{\text{e}}-n_{\text{ediv}}) \end{aligned} \quad (1.19)$$

$$\frac{\partial}{\partial t}n_{02}=\frac{1}{r}\frac{\partial}{\partial r}rD_{02}\frac{\partial}{\partial r}n_{02}-\frac{1}{Z_{\text{i}}}\nu_{\text{ionize}}\frac{n_{\text{e}}}{n_{01}+n_{02}+n_{03}}n_{02}+\nu_{\text{CX}}\frac{n_{\text{i}}}{n_{01}+n_{02}}n_{01} \quad (1.20)$$

$$\frac{\partial}{\partial t}n_{03}=\frac{1}{r}\frac{\partial}{\partial r}rD_{03}\frac{\partial}{\partial r}n_{03}-\frac{1}{Z_{\text{i}}}\nu_{\text{ionize}}\frac{n_{\text{e}}}{n_{01}+n_{02}+n_{03}}n_{03}+f_{\text{CX}}\frac{P_{\text{b}}}{E_{\text{b}}} \quad (1.21)$$

$$\frac{\partial}{\partial t}n_{\text{b}}^{\text{rp}}=\frac{P_{\text{b}\perp}}{E_{\text{b}}}+\nu_{\text{eff}}(\sqrt{\delta}n_{\text{b}}-n_{\text{b}}^{\text{rp}})-\nu_{\text{b}}n_{\text{b}}^{\text{rp}}-\frac{1}{r}\frac{\partial}{\partial r}\left(ru_{\text{b}}^{\text{rp}}n_{\text{b}}^{\text{rp}}-rD_{\text{b}}^{\text{rp}}\frac{\partial}{\partial r}n_{\text{b}}^{\text{rp}}\right) \quad (1.22)$$

2 Relations Between Fields and Potentials

$$E_r=-\frac{\partial}{\partial r}\phi \quad (2.1)$$

$$E_{\theta}=-\frac{\partial}{\partial t}A_{\theta}=-\dot{A}_{\theta} \quad (2.2)$$

$$E_{\phi}=-\frac{\partial}{\partial t}A_{\phi}=-\dot{A}_{\phi} \quad (2.3)$$

$$B_\theta = -\frac{\partial}{\partial r} A_\phi \quad (2.4)$$

$$B_\phi = \frac{1}{r} \frac{\partial}{\partial r} r A_\theta \quad (2.5)$$

3 Equations in $s(=r^2)$ coordinates

$$0 = 4 \frac{\partial}{\partial s} s \frac{\partial}{\partial s} \phi + \frac{1}{\epsilon_0} (-en_e + Z_i en_i + Z_b en_b + Z_b eg n_b^{\text{rp}}) \quad (3.1)$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} r \dot{A}_\theta = \left(4 \frac{\partial}{\partial s} s \frac{\partial}{\partial s} r A_\theta - 4 \frac{\partial}{\partial s} r A_\theta \right) + \mu_0 (-ern_e u_{e\theta} + Z_i ern_i u_{i\theta} + Z_b ern_b u_{b\theta}) \quad (3.2)$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} \dot{A}_\phi = 4 \frac{\partial}{\partial s} s \frac{\partial}{\partial s} A_\phi + \mu_0 (-en_e u_{e\phi} + Z_i en_i u_{i\phi} + Z_b en_b u_{b\phi}) \quad (3.3)$$

$$\frac{\partial}{\partial t} A_\phi = \dot{A}_\phi \quad (3.4)$$

$$\frac{\partial}{\partial t} r A_\theta = r \dot{A}_\theta \quad (3.5)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e = & -2 \frac{\partial}{\partial s} r n_e u_{er} + \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) - \nu_L (n_e - n_{\text{ediv}}) \\ & + (1 - f_{\text{CX}}) \frac{P_b}{E_b} \end{aligned} \quad (3.6)$$

$$\begin{aligned} \frac{\partial}{\partial t} r n_e u_{er} = & -2r \frac{\partial}{\partial s} u_{er} r n_e u_{er} + \frac{u_{e\theta}}{r} r n_e u_{e\theta} - 2s \frac{\partial}{\partial s} \frac{n_e T_e}{m_e} \\ & + 2 \frac{e}{m_e} s n_e \frac{\partial}{\partial s} \phi - \frac{e}{m_e} B_\phi r n_e u_{e\theta} - 2 \frac{e}{m_e} s \frac{\partial A_\phi}{\partial s} n_e u_{e\phi} \end{aligned} \quad (3.7)$$

$$\begin{aligned} \frac{\partial}{\partial t} r n_e u_{e\theta} = & -2 \frac{\partial}{\partial s} r u_{er} r n_e u_{e\theta} + \left(4 \frac{\partial}{\partial s} s \mu_e \frac{\partial}{\partial s} r n_e u_{e\theta} - 4 \frac{\partial}{\partial s} s \mu_e r u_{e\theta} \frac{\partial n_e}{\partial s} - 4 \frac{\partial}{\partial s} \mu_e r n_e u_{e\theta} \right) \\ & + \frac{e}{m_e} n_e r \dot{A}_\theta + \frac{e}{m_e} B_\phi r n_e u_{er} - \nu_{\text{NCe}} r n_e u_{e\theta} - \nu_{\text{ei1}} r n_e (u_{e\theta} - u_{i\theta}) + 2s \frac{\partial A_\phi}{\partial s} \frac{\nu_{\text{ei2}}}{B_\theta} n_e (u_{e\phi} - u_{i\phi}) \\ & - \frac{m_b}{m_e} \nu_{\text{be1}} r n_b (u_{e\theta} - u_{b\theta}) + 2 \frac{m_b}{m_e} s \frac{\partial A_\phi}{\partial s} \frac{\nu_{\text{be2}}}{B_\theta} n_e (u_{e\phi} - u_{b\phi}) \\ & - \frac{e^2 B_\phi^2 D_e}{m_e T_e} n_e \left[(r u_{e\theta} - r u_{i\theta}) + 2 \frac{\partial A_\phi}{\partial s} \frac{s}{B_\phi} (u_{e\phi} - u_{i\phi}) + s \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e \right. \\ & \left. + 2s \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial s} T_e - 2s \frac{1}{B_\phi} \frac{\partial}{\partial s} \phi \right] - 2\nu_L r n_e u_{e\theta} - \nu_{0e} r n_e u_{e\theta} \end{aligned}$$

$$+ 2\mu_{e2} \frac{B_\phi}{B_\theta^2} \frac{s}{m_e n_e} \frac{\partial}{\partial s} n_e T_e - 2\mu_{e2} \frac{B_\phi}{B_\theta^2} \frac{s}{m_e n_e^2} \frac{\partial n_e}{\partial s} n_e T_e \quad (3.8)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e u_{e\phi} = & -2 \frac{\partial}{\partial s} r u_{er} n_e u_{e\phi} + \left(4 \frac{\partial}{\partial s} s \mu_e \frac{\partial}{\partial s} n_e u_{e\phi} - 4 \frac{\partial}{\partial s} s \mu_e u_{e\phi} \frac{\partial n_e}{\partial s} \right) \\ & + \frac{e}{m_e} n_e \dot{A}_\phi + 2 \frac{e}{m_e} \frac{\partial A_\phi}{\partial s} r n_e u_{er} - \nu_{ei3} n_e (u_{e\phi} - u_{i\phi}) + 2 \frac{\partial A_\phi}{\partial s} \frac{\nu_{ei2}}{B_\theta} r n_e (u_{e\theta} - u_{i\theta}) \\ & + 2 \frac{m_b}{m_e} \frac{\partial A_\phi}{\partial s} \frac{\nu_{be2}}{B_\theta} r n_e (u_{e\theta} - u_{i\theta}) - \frac{m_b}{m_e} \nu_{be3} n_b (u_{e\phi} - u_{b\phi}) \\ & - 2 \frac{e^2 B_\phi D_e}{m_e T_e} \frac{\partial A_\phi}{\partial s} n_e \left[(r u_{e\theta} - r u_{i\theta}) + s \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e + 2s \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial s} T_e - 2s \frac{1}{B_\phi} \frac{\partial}{\partial s} \phi \right] \\ & - \frac{e^2 B_\phi^2 D_e}{m_e T_e} \left(\frac{B_\theta}{B_\phi} \right)^2 n_e (u_{e\phi} - u_{i\phi}) - 2\nu_L n_e u_{e\phi} - \nu_{0e} n_e u_{e\phi} \end{aligned} \quad (3.9)$$

$$\begin{aligned} \frac{\partial}{\partial t} \frac{3}{2} n_e T_e = & -5 \frac{\partial}{\partial s} r u_{er} n_e T_e + \left(4 \frac{\partial}{\partial s} s \chi_e \frac{\partial}{\partial s} n_e T_e - 4 \frac{\partial}{\partial s} s \chi_e T_e \frac{\partial n_e}{\partial s} \right) + 2 r u_{er} \frac{\partial}{\partial s} n_e T_e \\ & - e \frac{E_\theta}{r} r n_e u_{e\theta} - e E_\phi n_e u_{e\phi} - \frac{3}{2} \nu_{Tei} n_e (T_e - T_i) \\ & - \nu_L T_e (n_e - n_{ediv}) - \frac{3}{2} \nu_{LT_e} n_e (T_e - T_{ediv}) \\ & - E_H \nu_{ionize} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) + f_{ei} P_b + P_{\alpha e} + P_{RFe} - P_{Br} \end{aligned} \quad (3.10)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_i = & -2 \frac{\partial}{\partial s} r n_i u_{ir} + \frac{\nu_{ionize}}{Z_i} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) - \frac{\nu_L}{Z_i} (n_e - n_{ediv}) \\ & + \nu_b n_b + \nu_b g n_b^{rp} - f_{CX} \frac{P_b}{E_b} + \nu_{bL} n_b + S_{LC} - \nu_{OL} n_i \end{aligned} \quad (3.11)$$

$$\begin{aligned} \frac{\partial}{\partial t} r n_i u_{ir} = & -2r \frac{\partial}{\partial s} u_{ir} r n_i u_{ir} + \frac{u_{e\theta}}{r} r n_i u_{i\theta} - 2s \frac{\partial}{\partial s} \frac{n_i T_i}{m_i} \\ & - 2Z_i \frac{e}{m_i} s n_i \frac{\partial}{\partial s} \phi + Z_i \frac{e}{m_i} B_\phi r n_i u_{i\theta} + 2Z_i \frac{e}{m_i} s \frac{\partial A_\phi}{\partial s} n_i u_{i\phi} \end{aligned} \quad (3.12)$$

$$\begin{aligned} \frac{\partial}{\partial t} r n_i u_{i\theta} = & -2 \frac{\partial}{\partial s} r u_{ir} r n_i u_{i\theta} + \left(4 \frac{\partial}{\partial s} s \mu_i \frac{\partial}{\partial s} r n_i u_{i\theta} - 4 \frac{\partial}{\partial s} s \mu_i r u_{i\theta} \frac{\partial n_i}{\partial s} - 4 \frac{\partial}{\partial s} \mu_i r n_i u_{i\theta} \right) \\ & - Z_i \frac{e}{m_i} n_i r \dot{A}_\theta - Z_i \frac{e}{m_i} B_\phi r n_i u_{ir} - \nu_{NCi} r n_i u_{i\theta} - \frac{m_e}{m_i} \nu_{ei1} r n_e (u_{i\theta} - u_{e\theta}) \\ & + 2 \frac{m_e}{m_i} s \frac{\partial A_\phi}{\partial s} \frac{\nu_{ei2}}{B_\theta} n_e (u_{i\phi} - u_{e\phi}) - \frac{m_b}{m_i} \nu_{bi} r n_b (u_{i\theta} - u_{b\theta}) \\ & + \frac{e^2 B_\phi^2 D_e}{m_i T_e} n_e \left[(r u_{e\theta} - r u_{i\theta}) + 2 \frac{\partial A_\phi}{\partial s} \frac{s}{B_\phi} (u_{e\phi} - u_{i\phi}) + s \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e \right] \end{aligned}$$

$$\begin{aligned}
& + 2s \frac{1 - \alpha_e}{eB_\phi} \frac{\partial}{\partial s} T_e - 2s \frac{1}{B_\phi} \frac{\partial}{\partial s} \phi \Big] - 2\nu_L r n_i u_{i\theta} - \nu_{0i} r n_i u_{i\theta} - \nu_{CX} r n_i u_{i\theta} \\
& + r S_{LC\theta} - \nu_{OL} r n_i u_{i\theta} - 2\mu_{i2} \frac{B_\phi}{B_\theta^2} \frac{s}{m_i Z_i e n_i} \frac{\partial}{\partial s} n_i T_i + 2\mu_{i2} \frac{B_\phi}{B_\theta^2} \frac{s}{m_i Z_i e n_i^2} \frac{\partial n_i}{\partial s} n_i T_i + T_{\theta,i} \quad (3.13)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial t} n_i u_{i\phi} &= -2 \frac{\partial}{\partial s} r u_{ir} n_i u_{i\phi} + \left(4 \frac{\partial}{\partial s} s \mu_i \frac{\partial}{\partial s} n_i u_{i\phi} - 4 \frac{\partial}{\partial s} s \mu_i u_{i\phi} \frac{\partial n_i}{\partial s} \right) - Z_i \frac{e}{m_i} n_i \dot{A}_\phi - 2Z_i \frac{e}{m_i} \frac{\partial A_\phi}{\partial s} r n_i u_{ir} \\
& - \frac{m_e}{m_i} \nu_{ei3} n_e (u_{i\phi} - u_{e\phi}) + 2 \frac{m_e}{m_i} \frac{\partial A_\phi}{\partial s} \frac{\nu_{ei2}}{B_\theta} r n_e (u_{i\theta} - u_{e\theta}) - \frac{m_b}{m_i} \nu_{bi} n_b (u_{i\phi} - u_{b\phi}) \\
& + 2 \frac{e^2 B_\phi D_e}{m_i T_e} \frac{\partial A_\phi}{\partial s} n_e \left[(r u_{e\theta} - r u_{i\theta}) + s \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e + 2s \frac{1 - \alpha_e}{eB_\phi} \frac{\partial}{\partial s} T_e - 2s \frac{1}{B_\phi} \frac{\partial}{\partial s} \phi \right] \\
& + \frac{e^2 B_\phi^2 D_e}{m_i T_e} \left(\frac{B_\theta}{B_\phi} \right)^2 n_e (u_{e\phi} - u_{i\phi}) - 2\nu_L n_i u_{i\phi} - \nu_{0i} n_i u_{i\phi} - \nu_{CX} n_i u_{i\phi} \\
& + S_{LC\phi} - \nu_{OL} n_i u_{i\phi} + T_{\phi,i} \quad (3.14)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial t} \frac{3}{2} n_i T_i &= -5 \frac{\partial}{\partial s} r u_{er} n_e T_e + \left(4 \frac{\partial}{\partial s} s \chi_e \frac{\partial}{\partial s} n_i T_i - 4 \frac{\partial}{\partial s} s \chi_i T_i \frac{\partial n_i}{\partial s} \right) + 2r u_{ir} \frac{\partial}{\partial s} n_i T_i \\
& + Z_i e \frac{E_\theta}{r} r n_i u_{i\theta} + Z_i e E_\phi n_i u_{i\phi} - \frac{3}{2} \nu_{Tei} n_e (T_i - T_e) + m_b v_b \frac{B_\theta u_{i\theta} + B_\phi u_{i\phi}}{B} \frac{P_{b\parallel}}{E_b} \\
& - \frac{\nu_L}{Z_i} T_i (n_e - n_{ediv}) - \frac{3}{2} \nu_{LT_i} n_i (T_i - T_{idiv}) \\
& + \frac{3}{2} \frac{\nu_{ionize}}{Z_i} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} T_{01} + n_{02} T_{02} + n_{03} T_{03}) - \frac{3}{2} \nu_{CX} n_i (T_i - T_{01}) \\
& + (1 - f_{ei}) P_b + P_{\alpha i} + P_{RFi} \quad (3.15)
\end{aligned}$$

$$\frac{\partial}{\partial t} n_b = \frac{P_b}{E_b} - g \frac{P_{b\perp}}{E_b} - \nu_b n_b - \nu_{bL} n_b - \nu_{eff} g (\sqrt{\delta} n_b - n_b^{rp}) + 4 \frac{\partial}{\partial s} s f_t D_b \frac{\partial}{\partial s} n_b \quad (3.16)$$

$$\begin{aligned}
\frac{\partial}{\partial t} r n_b u_{b\theta} &= -Z_b \frac{e}{m_b} n_b r \dot{A}_\theta - \nu_{be1} r n_b (u_{b\theta} - u_{e\theta}) + 2s \frac{\partial A_\phi}{\partial s} \frac{\nu_{be2}}{B_\theta} n_b (u_{b\phi} - u_{e\phi}) - \nu_{bi} r n_b (u_{b\theta} - u_{i\theta}) \\
& - \nu_{0b} r n_b u_{b\theta} - \nu_{CX} r n_b u_{b\theta} + r v_{b\theta} \frac{P_{b\parallel}}{E_b} - \nu_{bL} r n_b u_{b\theta} - \nu_{eff} g \sqrt{\delta} r n_b u_{b\theta} \\
& + 4 \frac{\partial}{\partial s} s f_t D_b \frac{\partial}{\partial s} r n_b u_{b\theta} - 4 \frac{\partial}{\partial s} \left(s f_t D_b \frac{\partial r u_{b\theta}}{\partial s} \right) n_b \quad (3.17)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial t} n_b u_{b\phi} &= -Z_b \frac{e}{m_b} n_b \dot{A}_\phi - \nu_{be3} n_b (u_{b\phi} - u_{e\phi}) + 2 \frac{\partial A_\phi}{\partial s} \frac{\nu_{be2}}{B_\theta} n_b (u_{b\theta} - u_{e\theta}) - \nu_{bi} n_b (u_{b\phi} - u_{i\phi}) \\
& - \nu_{0b} n_b u_{b\phi} - \nu_{CX} n_b u_{b\phi} + v_{b\phi} \frac{P_{b\parallel}}{E_b} - \nu_{bL} n_b u_{b\phi} - \nu_{eff} g \sqrt{\delta} n_b u_{b\phi}
\end{aligned}$$

$$+ 4 \frac{\partial}{\partial s} s f_t D_b \frac{\partial}{\partial s} n_b u_{b\phi} - 4 \frac{\partial}{\partial s} \left(s f_t D_b \frac{\partial u_{b\phi}}{\partial s} \right) n_b \quad (3.18)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_{01} = & 4 \frac{\partial}{\partial s} s D_{01} \frac{\partial}{\partial s} n_{01} - \frac{1}{Z_i} \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} n_{01} - \nu_{\text{CX}} \frac{n_i}{n_{01} + n_{02}} n_{01} \\ & + \gamma_0 \frac{\nu_L}{Z_i} (n_e - n_{\text{ediv}}) \end{aligned} \quad (3.19)$$

$$\frac{\partial}{\partial t} n_{02} = 4 \frac{\partial}{\partial s} s D_{02} \frac{\partial}{\partial s} n_{02} - \frac{1}{Z_i} \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} n_{02} + \nu_{\text{CX}} \frac{n_i}{n_{01} + n_{02}} n_{01} \quad (3.20)$$

$$\frac{\partial}{\partial t} n_{03} = 4 \frac{\partial}{\partial s} s D_{03} \frac{\partial}{\partial s} n_{03} - \frac{1}{Z_i} \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} n_{03} + f_{\text{CX}} \frac{P_b}{E_b} \quad (3.21)$$

$$\frac{\partial}{\partial t} n_b^{\text{rp}} = \frac{P_{b\perp}}{E_b} + \nu_{\text{eff}} (\sqrt{\delta} n_b - n_b^{\text{rp}}) - \nu_b n_b^{\text{rp}} - 2 \frac{\partial}{\partial s} r u_b^{\text{rp}} n_b^{\text{rp}} + 4 \frac{\partial}{\partial s} s D_b^{\text{rp}} \frac{\partial}{\partial s} n_b^{\text{rp}} \quad (3.22)$$

4 Boundary Conditions

num.	name	variable	center	edge
1	LQm1	ϕ	N	0
2	LQm2	$r\dot{A}_\theta$	0	$2s_b B_V$
3	LQm3	\dot{A}_ϕ	N	$-2bB_{\theta b}$
4	LQm4	A_ϕ	\times	\times
5	LQm5	rA_θ	\times	\times
6	LQe1	n_e	\times	$\times(N)$
7	LQe2	$rn_e u_{er}$	0	$0(\times)$
8	LQe3	$rn_e u_{e\theta}$	0	0
9	LQe4	$n_e u_{e\phi}$	N	0
10	LQe5	$n_e T_e$	N	N
11	LQi1	n_i	\times	$\times(N)$
12	LQi2	$rn_i u_{ir}$	0	$0(\times)$
13	LQi3	$rn_i u_{i\theta}$	0	0
14	LQi4	$n_i u_{i\phi}$	N	0
15	LQi5	$n_i T_i$	N	N
16	LQb1	n_b	$\times(N)$	$\times(N)$
17	LQb3	$rn_b u_{b\theta}$	$\times(0)$	$\times(0)$
18	LQb4	$n_b u_{b\phi}$	$\times(0)$	$\times(0)$
19	LQn1	n_{01}	N	$2bS_{\text{gas}}$
20	LQn2	n_{02}	N	0
20	LQn3	n_{03}	N	0
21	LQr1	n_b^{rp}	N	N

0: Dirichlet condition ($u = 0$)

N: Neumann condition ($u' = 0$)

value: Neumann condition ($u' = \text{value}$)

\times : no condition