1 Equations in r coordinates

$$0 = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\phi + \frac{1}{\epsilon_0}\left(-e n_\mathrm{e} + Z_\mathrm{i} e n_\mathrm{i} + Z_\mathrm{b} e n_\mathrm{b} + Z_\mathrm{b} e g n_\mathrm{b}^\mathrm{rp}\right) \tag{1.1}$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\dot{A}_\theta = \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} r A_\theta + \mu_0(-e n_\mathrm{e} u_{\mathrm{e}\theta} + Z_\mathrm{i} e n_\mathrm{i} u_{\mathrm{i}\theta} + Z_\mathrm{b} e n_\mathrm{b} u_{\mathrm{b}\theta}) \tag{1.2}$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\dot{A}_\phi = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} A_\phi + \mu_0(-e n_\mathrm{e} u_{\mathrm{e}\phi} + Z_\mathrm{i} e n_\mathrm{i} u_{\mathrm{i}\phi} + Z_\mathrm{b} e n_\mathrm{b} u_{\mathrm{b}\phi}) \tag{1.3}$$

$$\frac{\partial}{\partial t} A_\phi = \dot{A}_\phi \tag{1.4}$$

$$\frac{\partial}{\partial t} A_\theta = \dot{A}_\theta \tag{1.5}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_\mathrm{e} = &-\frac{1}{r}\frac{\partial}{\partial r} r n_\mathrm{e} u_{\mathrm{e}r} + \nu_\mathrm{ionize}\frac{n_\mathrm{e}}{n_{01}+n_{02}+n_{03}}(n_{01}+n_{02}+n_{03}) - \nu_\mathrm{L}(n_\mathrm{e} - n_\mathrm{ediv}) \\ &+ (1-f_\mathrm{CX})\frac{P_\mathrm{b}}{E_\mathrm{b}}\end{aligned} \tag{1.6}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_\mathrm{e} u_{\mathrm{e}r} = &-\frac{1}{r}\frac{\partial}{\partial r} r u_{\mathrm{e}r} n_\mathrm{e} u_{\mathrm{e}r} + \frac{1}{r} u_{\mathrm{e}\theta} n_\mathrm{e} u_{\mathrm{e}\theta} - \frac{\partial}{\partial r}\frac{n_\mathrm{e} T_\mathrm{e}}{m_\mathrm{e}} \\ &- \frac{e}{m_\mathrm{e}} n_\mathrm{e} E_r - \frac{e}{m_\mathrm{e}} B_\phi n_\mathrm{e} u_{\mathrm{e}\theta} + \frac{e}{m_\mathrm{e}} B_\theta n_\mathrm{e} u_{\mathrm{e}\phi}\end{aligned} \tag{1.7}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_\mathrm{e} u_{\mathrm{e}\theta} = &-\frac{1}{r^2}\frac{\partial}{\partial r} r^2 u_{\mathrm{e}r} n_\mathrm{e} u_{\mathrm{e}\theta} + \frac{1}{r^2}\frac{\partial}{\partial r} r^3 n_\mathrm{e}\mu_\mathrm{e}\frac{\partial}{\partial r}\frac{u_{\mathrm{e}\theta}}{r} - \frac{e}{m_\mathrm{e}} n_\mathrm{e} E_\theta + \frac{e}{m_\mathrm{e}} B_\phi n_\mathrm{e} u_{\mathrm{e}r} \\ &- \nu_\mathrm{NCe} n_\mathrm{e} u_{\mathrm{e}\theta} - \nu_{\mathrm{ei}1} n_\mathrm{e}(u_{\mathrm{e}\theta} - u_{\mathrm{i}\theta}) - \nu_{\mathrm{ei}2} n_\mathrm{e}(u_{\mathrm{e}\phi} - u_{\mathrm{i}\phi}) \\ &- \frac{m_\mathrm{b}}{m_\mathrm{e}}\nu_{\mathrm{be}1} n_\mathrm{b}(u_{\mathrm{e}\theta} - u_{\mathrm{b}\theta}) - \frac{m_\mathrm{b}}{m_\mathrm{e}}\nu_{\mathrm{be}2} n_\mathrm{b}(u_{\mathrm{e}\phi} - u_{\mathrm{b}\phi}) \\ &- \frac{e^2 B_\phi^2 D_\mathrm{e}}{m_\mathrm{e} T_\mathrm{e}} n_\mathrm{e}\left[u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi} u_{\mathrm{e}\phi} + \frac{B}{B_\phi}\left\langle\frac{\omega}{m}\right\rangle_\mathrm{e} r + \frac{1-\alpha_\mathrm{e}}{e B_\phi}\frac{\partial}{\partial r} T_\mathrm{e} + \frac{E_r}{B_\phi}\right] \\ &- 2\nu_\mathrm{L} n_\mathrm{e} u_{\mathrm{e}\theta} - \nu_{0\mathrm{e}} n_\mathrm{e} u_{\mathrm{e}\theta} + F_{\mathrm{e}\theta}^\mathrm{neo}\end{aligned} \tag{1.8}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_\mathrm{e} u_{\mathrm{e}\phi} = &-\frac{1}{r}\frac{\partial}{\partial r} r u_{\mathrm{e}r} n_\mathrm{e} u_{\mathrm{e}\phi} + \frac{1}{r}\frac{\partial}{\partial r} r n_\mathrm{e}\mu_\mathrm{e}\frac{\partial}{\partial r} u_{\mathrm{e}\phi} - \frac{e}{m_\mathrm{e}} n_\mathrm{e} E_\phi - \frac{e}{m_\mathrm{e}} B_\theta n_\mathrm{e} u_{\mathrm{e}r} \\ &- \nu_{\mathrm{ei}3} n_\mathrm{e}(u_{\mathrm{e}\phi} - u_{\mathrm{i}\phi}) - \nu_{\mathrm{ei}2} n_\mathrm{e}(u_{\mathrm{e}\theta} - u_{\mathrm{i}\theta}) - \frac{m_\mathrm{b}}{m_\mathrm{e}}\nu_{\mathrm{be}2} n_\mathrm{b}(u_{\mathrm{e}\theta} - u_{\mathrm{b}\theta}) - \frac{m_\mathrm{b}}{m_\mathrm{e}}\nu_{\mathrm{be}3} n_\mathrm{b}(u_{\mathrm{e}\phi} - u_{\mathrm{b}\phi}) \\ &+ \frac{e^2 B_\phi^2 D_\mathrm{e}}{m_\mathrm{e} T_\mathrm{e}}\frac{B_\theta}{B_\phi} n_\mathrm{e}\left[u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi} u_{\mathrm{e}\phi} + \frac{B}{B_\phi}\left\langle\frac{\omega}{m}\right\rangle_\mathrm{e} r + \frac{1-\alpha_\mathrm{e}}{e B_\phi}\frac{\partial}{\partial r} T_\mathrm{e} + \frac{E_r}{B_\phi}\right]\end{aligned}$$

$$- 2\nu_{\mathrm{L}} n_{\mathrm{e}} u_{\mathrm{e}\phi} - \nu_{0\mathrm{e}} n_{\mathrm{e}} u_{\mathrm{e}\phi} \tag{1.9}$$

$$\begin{aligned}\frac{\partial}{\partial t}\frac{3}{2} n_{\mathrm{e}} T_{\mathrm{e}} = &-\frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{5}{2} u_{\mathrm{e}r} n_{\mathrm{e}} T_{\mathrm{e}} - n_{\mathrm{e}} \chi_{\mathrm{e}} \frac{\partial}{\partial r} T_{\mathrm{e}} \right) + u_{\mathrm{e}r} \frac{\partial}{\partial r} n_{\mathrm{e}} T_{\mathrm{e}} - e E_\theta n_{\mathrm{e}} u_{\mathrm{e}\theta} - e E_\phi n_{\mathrm{e}} u_{\mathrm{e}\phi} \\ &- \frac{3}{2} \nu_{\mathrm{Tei}} n_{\mathrm{e}} (T_{\mathrm{e}} - T_{\mathrm{i}}) - \nu_{\mathrm{L}} T_{\mathrm{e}} (n_{\mathrm{e}} - n_{\mathrm{ediv}}) - \frac{3}{2} \nu_{\mathrm{LT_e}} n_{\mathrm{e}} (T_{\mathrm{e}} - T_{\mathrm{ediv}}) \\ &- E_{\mathrm{H}} \nu_{\mathrm{ionize}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) + f_{\mathrm{ei}} P_{\mathrm{b}} + P_{\alpha\mathrm{e}} + P_{\mathrm{RFe}} - P_{\mathrm{Br}}\end{aligned} \tag{1.10}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_{\mathrm{i}} = &-\frac{1}{r}\frac{\partial}{\partial r} r n_{\mathrm{i}} u_{\mathrm{i}r} + \frac{\nu_{\mathrm{ionize}}}{Z_{\mathrm{i}}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) - \frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}} (n_{\mathrm{e}} - n_{\mathrm{ediv}}) \\ &+ \nu_{\mathrm{b}} n_{\mathrm{b}} + \nu_{\mathrm{b}} g n_{\mathrm{b}}^{\mathrm{rp}} - f_{\mathrm{CX}} \frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} + \nu_{\mathrm{bL}} n_{\mathrm{b}} + S_{\mathrm{LC}} - \nu_{\mathrm{OL}} n_{\mathrm{i}}\end{aligned} \tag{1.11}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_{\mathrm{i}} u_{\mathrm{i}r} = &-\frac{1}{r}\frac{\partial}{\partial r} r u_{\mathrm{i}r} n_{\mathrm{i}} u_{\mathrm{i}r} + \frac{1}{r} u_{\mathrm{i}\theta} n_{\mathrm{i}} u_{\mathrm{i}\theta} - \frac{\partial}{\partial r} \frac{n_{\mathrm{i}} T_{\mathrm{i}}}{m_{\mathrm{i}}} \\ &+ Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} n_{\mathrm{i}} E_r + Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} B_\phi n_{\mathrm{i}} u_{\mathrm{i}\theta} - Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} B_\theta n_{\mathrm{i}} u_{\mathrm{i}\phi}\end{aligned} \tag{1.12}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_{\mathrm{i}} u_{\mathrm{i}\theta} = &-\frac{1}{r^2}\frac{\partial}{\partial r} r^2 u_{\mathrm{i}r} n_{\mathrm{i}} u_{\mathrm{i}\theta} + \frac{1}{r^2}\frac{\partial}{\partial r} r^3 n_{\mathrm{i}} \mu_{\mathrm{i}} \frac{\partial}{\partial r} \frac{u_{\mathrm{i}\theta}}{r} + Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} n_{\mathrm{i}} E_\theta - Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} B_\phi n_{\mathrm{i}} u_{\mathrm{i}r} \\ &- \nu_{\mathrm{NCi}} n_{\mathrm{i}} u_{\mathrm{i}\theta} - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} \nu_{\mathrm{ei1}} n_{\mathrm{e}} (u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta}) - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} \nu_{\mathrm{ei2}} n_{\mathrm{e}} (u_{\mathrm{i}\phi} - u_{\mathrm{e}\phi}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{i}}} \nu_{\mathrm{bi}} n_{\mathrm{b}} (u_{\mathrm{i}\theta} - u_{\mathrm{b}\theta}) \\ &+ \frac{e^2 B_\phi^2 D_{\mathrm{e}}}{m_{\mathrm{i}} T_{\mathrm{e}}} n_{\mathrm{e}} \left[u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi} u_{\mathrm{e}\phi} + \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_{\mathrm{e}} r + \frac{1 - \alpha_{\mathrm{e}}}{e B_\phi} \frac{\partial}{\partial r} T_{\mathrm{e}} + \frac{E_r}{B_\phi} \right] \\ &- 2\nu_{\mathrm{L}} n_{\mathrm{i}} u_{\mathrm{i}\theta} - \nu_{0\mathrm{i}} n_{\mathrm{i}} u_{\mathrm{i}\theta} - \nu_{\mathrm{CX}} n_{\mathrm{i}} u_{\mathrm{i}\theta} + S_{\mathrm{LC}\theta} - \nu_{\mathrm{OL}} n_{\mathrm{i}} u_{\mathrm{i}\theta} + F_{\mathrm{i}\theta}^{\mathrm{neo}} + T_{\mathrm{i}\theta}\end{aligned} \tag{1.13}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_{\mathrm{i}} u_{\mathrm{i}\phi} = &-\frac{1}{r}\frac{\partial}{\partial r} r u_{\mathrm{i}r} n_{\mathrm{i}} u_{\mathrm{i}\phi} + \frac{1}{r}\frac{\partial}{\partial r} r n_{\mathrm{i}} \mu_{\mathrm{i}} \frac{\partial}{\partial r} u_{\mathrm{i}\phi} + Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} n_{\mathrm{i}} E_\phi + Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} B_\theta n_{\mathrm{i}} u_{\mathrm{i}r} \\ &- \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} \nu_{\mathrm{ei3}} n_{\mathrm{e}} (u_{\mathrm{i}\phi} - u_{\mathrm{e}\phi}) - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} \nu_{\mathrm{ei2}} n_{\mathrm{e}} (u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{i}}} \nu_{\mathrm{bi}} n_{\mathrm{b}} (u_{\mathrm{i}\phi} - u_{\mathrm{b}\phi}) \\ &- \frac{e^2 B_\phi^2 D_{\mathrm{e}}}{m_{\mathrm{i}} T_{\mathrm{e}}} \frac{B_\theta}{B_\phi} n_{\mathrm{e}} \left[u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi} u_{\mathrm{e}\phi} + \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_{\mathrm{e}} r + \frac{1 - \alpha_{\mathrm{e}}}{e B_\phi} \frac{\partial}{\partial r} T_{\mathrm{e}} + \frac{E_r}{B_\phi} \right] \\ &- 2\nu_{\mathrm{L}} n_{\mathrm{i}} u_{\mathrm{i}\phi} - \nu_{0\mathrm{i}} n_{\mathrm{i}} u_{\mathrm{i}\phi} - \nu_{\mathrm{CX}} n_{\mathrm{i}} u_{\mathrm{i}\phi} + S_{\mathrm{LC}\phi} - \nu_{\mathrm{OL}} n_{\mathrm{i}} u_{\mathrm{i}\phi} + T_{\mathrm{i}\phi}\end{aligned} \tag{1.14}$$

$$\begin{aligned}\frac{\partial}{\partial t}\frac{3}{2} n_{\mathrm{i}} T_{\mathrm{i}} = &-\frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{5}{2} u_{\mathrm{i}r} n_{\mathrm{i}} T_{\mathrm{i}} - n_{\mathrm{i}} \chi_{\mathrm{i}} \frac{\partial}{\partial r} T_{\mathrm{i}} \right) + u_{\mathrm{i}r} \frac{\partial}{\partial r} n_{\mathrm{i}} T_{\mathrm{i}} + Z_{\mathrm{i}} e E_\theta n_{\mathrm{i}} u_{\mathrm{i}\theta} + Z_{\mathrm{i}} e E_\phi n_{\mathrm{i}} u_{\mathrm{i}\phi} \\ &- \frac{3}{2} \nu_{\mathrm{Tei}} n_{\mathrm{e}} (T_{\mathrm{i}} - T_{\mathrm{e}}) + m_{\mathrm{b}} v_{\mathrm{b}} \frac{B_\theta u_{\mathrm{i}\theta} + B_\phi u_{\mathrm{i}\phi}}{B} \frac{P_{\mathrm{b}\parallel}}{E_{\mathrm{b}}} - \frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}} T_{\mathrm{i}} (n_{\mathrm{e}} - n_{\mathrm{ediv}}) \\ &- \frac{3}{2} \nu_{\mathrm{LT_i}} n_{\mathrm{i}} (T_{\mathrm{i}} - T_{\mathrm{idiv}}) + \frac{3}{2} \frac{\nu_{\mathrm{ionize}}}{Z_{\mathrm{i}}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} (n_{01} T_{01} + n_{02} T_{02} + n_{03} T_{03})\end{aligned}$$

$$- \frac{3}{2}\nu_{\mathrm{CX}} n_{\mathrm{i}}(T_{\mathrm{i}} - T_{01}) + (1 - f_{\mathrm{ei}})P_{\mathrm{b}} + P_{\alpha \mathrm{i}} + P_{\mathrm{RFi}} \tag{1.15}$$

$$\frac{\partial}{\partial t} n_{\mathrm{b}} = \frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} - g\frac{P_{\mathrm{b}\perp}}{E_{\mathrm{b}}} - \nu_{\mathrm{b}} n_{\mathrm{b}} - \nu_{\mathrm{bL}} n_{\mathrm{b}} - \nu_{\mathrm{eff}} g(\sqrt{\delta} n_{\mathrm{b}} - n_{\mathrm{b}}^{\mathrm{rp}}) + \frac{1}{r}\frac{\partial}{\partial r} r f_t D_{\mathrm{b}} \frac{\partial}{\partial r} n_{\mathrm{b}} \tag{1.16}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_{\mathrm{b}} u_{\mathrm{b}\theta} = {} & Z_{\mathrm{b}} \frac{e}{m_{\mathrm{b}}} n_{\mathrm{b}} E_\theta - \nu_{\mathrm{be1}} n_{\mathrm{b}}(u_{\mathrm{b}\theta} - u_{\mathrm{e}\theta}) - \nu_{\mathrm{be2}} n_{\mathrm{b}}(u_{\mathrm{b}\phi} - u_{\mathrm{e}\phi}) - \nu_{\mathrm{bi}} n_{\mathrm{b}}(u_{\mathrm{b}\theta} - u_{\mathrm{i}\theta}) \\ & - \nu_{0\mathrm{b}} n_{\mathrm{b}} u_{\mathrm{b}\theta} - \nu_{\mathrm{CX}} n_{\mathrm{b}} u_{\mathrm{b}\theta} + v_{\mathrm{b}\theta} \frac{P_{\mathrm{b}\parallel}}{E_{\mathrm{b}}} - \nu_{\mathrm{bL}} n_{\mathrm{b}} u_{\mathrm{b}\theta} - \nu_{\mathrm{eff}} g\sqrt{\delta} n_{\mathrm{b}} u_{\mathrm{b}\theta} \\ & + \frac{1}{r}\frac{\partial}{\partial r} r u_{\mathrm{b}\theta} f_t D_{\mathrm{b}} \frac{\partial}{\partial r} n_{\mathrm{b}}\end{aligned} \tag{1.17}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_{\mathrm{b}} u_{\mathrm{b}\phi} = {} & Z_{\mathrm{b}} \frac{e}{m_{\mathrm{b}}} n_{\mathrm{b}} E_\phi - \nu_{\mathrm{be3}} n_{\mathrm{b}}(u_{\mathrm{b}\phi} - u_{\mathrm{e}\phi}) - \nu_{\mathrm{be2}} n_{\mathrm{b}}(u_{\mathrm{b}\theta} - u_{\mathrm{e}\theta}) - \nu_{\mathrm{bi}} n_{\mathrm{b}}(u_{\mathrm{b}\phi} - u_{\mathrm{i}\phi}) \\ & - \nu_{0\mathrm{b}} n_{\mathrm{b}} u_{\mathrm{b}\phi} - \nu_{\mathrm{CX}} n_{\mathrm{b}} u_{\mathrm{b}\phi} + v_{\mathrm{b}\phi} \frac{P_{\mathrm{b}\parallel}}{E_{\mathrm{b}}} - \nu_{\mathrm{bL}} n_{\mathrm{b}} u_{\mathrm{b}\phi} - \nu_{\mathrm{eff}} g\sqrt{\delta} n_{\mathrm{b}} u_{\mathrm{b}\phi} \\ & + \frac{1}{r}\frac{\partial}{\partial r} r u_{\mathrm{b}\phi} f_t D_{\mathrm{b}} \frac{\partial}{\partial r} n_{\mathrm{b}}\end{aligned} \tag{1.18}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_{01} = {} & \frac{1}{r}\frac{\partial}{\partial r} r D_{01} \frac{\partial}{\partial r} n_{01} - \frac{1}{Z_{\mathrm{i}}} \nu_{\mathrm{ionize}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} n_{01} - \nu_{\mathrm{CX}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02}} n_{01} \\ & + \gamma_0 \frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}} (n_{\mathrm{e}} - n_{\mathrm{ediv}}) + \frac{1}{r}\frac{\partial}{\partial r} r \Gamma^{\mathrm{puff}} \Big|_{\mathrm{b}}\end{aligned} \tag{1.19}$$

$$\frac{\partial}{\partial t} n_{02} = \frac{1}{r}\frac{\partial}{\partial r} r D_{02} \frac{\partial}{\partial r} n_{02} - \frac{1}{Z_{\mathrm{i}}} \nu_{\mathrm{ionize}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} n_{02} + \nu_{\mathrm{CX}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02}} n_{01} \tag{1.20}$$

$$\frac{\partial}{\partial t} n_{03} = \frac{1}{r}\frac{\partial}{\partial r} r D_{03} \frac{\partial}{\partial r} n_{03} - \frac{1}{Z_{\mathrm{i}}} \nu_{\mathrm{ionize}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} n_{03} + f_{\mathrm{CX}} \frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} \tag{1.21}$$

$$\frac{\partial}{\partial t} n_{\mathrm{b}}^{\mathrm{rp}} = \frac{P_{\mathrm{b}\perp}}{E_{\mathrm{b}}} + \nu_{\mathrm{eff}}(\sqrt{\delta} n_{\mathrm{b}} - n_{\mathrm{b}}^{\mathrm{rp}}) - \nu_{\mathrm{b}} n_{\mathrm{b}}^{\mathrm{rp}} - \frac{1}{r}\frac{\partial}{\partial r}\left(r u_{\mathrm{b}}^{\mathrm{rp}} n_{\mathrm{b}}^{\mathrm{rp}} - r D_{\mathrm{b}}^{\mathrm{rp}} \frac{\partial}{\partial r} n_{\mathrm{b}}^{\mathrm{rp}} \right) \tag{1.22}$$

2 Relations Between Fields and Potentials

$$E_r = -\frac{\partial}{\partial r}\phi \tag{2.1}$$

$$E_\theta = -\frac{\partial}{\partial t} A_\theta = -\dot{A}_\theta \tag{2.2}$$

$$E_\phi = -\frac{\partial}{\partial t} A_\phi = -\dot{A}_\phi \tag{2.3}$$

$$B_\theta = -\frac{\partial}{\partial r} A_\phi \tag{2.4}$$

$$B_\phi = \frac{1}{r}\frac{\partial}{\partial r} r A_\theta \tag{2.5}$$

3 Equations in $s(=r^2)$ coordinates

$$0 = 4\frac{\partial}{\partial s} s \frac{\partial}{\partial s}\phi + \frac{1}{\epsilon_0}\left(-e n_\mathrm{e} + Z_\mathrm{i} e n_\mathrm{i} + Z_\mathrm{b} e n_\mathrm{b} + Z_\mathrm{b} e g n_\mathrm{b}^\mathrm{rp}\right) \tag{3.1}$$

$$\frac{1}{c^2}\frac{\partial}{\partial t} r\dot{A}_\theta = \left(4\frac{\partial}{\partial s} s \frac{\partial}{\partial s} r A_\theta - 4\frac{\partial}{\partial s} r A_\theta\right) + \mu_0(-e r n_\mathrm{e} u_{\mathrm{e}\theta} + Z_\mathrm{i} e r n_\mathrm{i} u_{\mathrm{i}\theta} + Z_\mathrm{b} e r n_\mathrm{b} u_{\mathrm{b}\theta}) \tag{3.2}$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\dot{A}_\phi = 4\frac{\partial}{\partial s} s \frac{\partial}{\partial s} A_\phi + \mu_0(-e n_\mathrm{e} u_{\mathrm{e}\phi} + Z_\mathrm{i} e n_\mathrm{i} u_{\mathrm{i}\phi} + Z_\mathrm{b} e n_\mathrm{b} u_{\mathrm{b}\phi}) \tag{3.3}$$

$$\frac{\partial}{\partial t} A_\phi = \dot{A}_\phi \tag{3.4}$$

$$\frac{\partial}{\partial t} r A_\theta = r\dot{A}_\theta \tag{3.5}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_\mathrm{e} = &-2\frac{\partial}{\partial s} r n_\mathrm{e} u_{\mathrm{e}r} + \nu_\mathrm{ionize}\frac{n_\mathrm{e}}{n_{01}+n_{02}+n_{03}}(n_{01}+n_{02}+n_{03}) - \nu_\mathrm{L}(n_\mathrm{e} - n_\mathrm{ediv}) \\ &+ (1 - f_\mathrm{CX})\frac{P_\mathrm{b}}{E_\mathrm{b}}\end{aligned} \tag{3.6}$$

$$\begin{aligned}\frac{\partial}{\partial t} r n_\mathrm{e} u_{\mathrm{e}r} = &-2r\frac{\partial}{\partial s} u_{\mathrm{e}r} r n_\mathrm{e} u_{\mathrm{e}r} + \frac{u_{\mathrm{e}\theta}}{r} r n_\mathrm{e} u_{\mathrm{e}\theta} - 2s\frac{\partial}{\partial s}\frac{n_\mathrm{e} T_\mathrm{e}}{m_\mathrm{e}} \\ &+ 2\frac{e}{m_\mathrm{e}} s n_\mathrm{e}\frac{\partial}{\partial s}\phi - \frac{e}{m_\mathrm{e}} B_\phi r n_\mathrm{e} u_{\mathrm{e}\theta} - 2\frac{e}{m_\mathrm{e}} s\frac{\partial A_\phi}{\partial s} n_\mathrm{e} u_{\mathrm{e}\phi}\end{aligned} \tag{3.7}$$

$$\begin{aligned}\frac{\partial}{\partial t} r n_\mathrm{e} u_{\mathrm{e}\theta} = &-2\frac{\partial}{\partial s} r u_{\mathrm{e}r} r n_\mathrm{e} u_{\mathrm{e}\theta} + \left(4\frac{\partial}{\partial s} s\mu_\mathrm{e}\frac{\partial}{\partial s} r n_\mathrm{e} u_{\mathrm{e}\theta} - 4\frac{\partial}{\partial s} s\mu_\mathrm{e} r u_{\mathrm{e}\theta}\frac{\partial n_\mathrm{e}}{\partial s} - 4\frac{\partial}{\partial s}\mu_\mathrm{e} r n_\mathrm{e} u_{\mathrm{e}\theta}\right) \\ &+ \frac{e}{m_\mathrm{e}} n_\mathrm{e} r\dot{A}_\theta + \frac{e}{m_\mathrm{e}} B_\phi r n_\mathrm{e} u_{\mathrm{e}r} - \nu_\mathrm{NCe} r n_\mathrm{e} u_{\mathrm{e}\theta} - \nu_{\mathrm{ei}1} r n_\mathrm{e}(u_{\mathrm{e}\theta} - u_{\mathrm{i}\theta}) + 2s\frac{\partial A_\phi}{\partial s}\frac{\nu_{\mathrm{ei}2}}{B_\theta} n_\mathrm{e}(u_{\mathrm{e}\phi} - u_{\mathrm{i}\phi}) \\ &- \frac{m_\mathrm{b}}{m_\mathrm{e}}\nu_{\mathrm{be}1} r n_\mathrm{b}(u_{\mathrm{e}\theta} - u_{\mathrm{b}\theta}) + 2\frac{m_\mathrm{b}}{m_\mathrm{e}} s\frac{\partial A_\phi}{\partial s}\frac{\nu_{\mathrm{be}2}}{B_\theta} n_\mathrm{e}(u_{\mathrm{e}\phi} - u_{\mathrm{b}\phi}) \\ &- \frac{e^2 B_\phi^2 D_\mathrm{e}}{m_\mathrm{e} T_\mathrm{e}} n_\mathrm{e}\left[r u_{\mathrm{e}\theta} + 2\frac{\partial A_\phi}{\partial s}\frac{s}{B_\phi} u_{\mathrm{e}\phi} + s\frac{B}{B_\phi}\left\langle\frac{\omega}{m}\right\rangle_\mathrm{e} + 2s\frac{1-\alpha_\mathrm{e}}{eB_\phi}\frac{\partial}{\partial s} T_\mathrm{e} - 2s\frac{1}{B_\phi}\frac{\partial}{\partial s}\phi\right] \\ &- 2\nu_\mathrm{L} r n_\mathrm{e} u_{\mathrm{e}\theta} - \nu_{0\mathrm{e}} r n_\mathrm{e} u_{\mathrm{e}\theta} + r F_{\mathrm{e}\theta}^\mathrm{neo}\end{aligned} \tag{3.8}$$

$$\frac{\partial}{\partial t}n_{\mathrm{e}}u_{\mathrm{e}\phi} = -2\frac{\partial}{\partial s}ru_{\mathrm{er}}n_{\mathrm{e}}u_{\mathrm{e}\phi} + \left(4\frac{\partial}{\partial s}s\mu_{\mathrm{e}}\frac{\partial}{\partial s}n_{\mathrm{e}}u_{\mathrm{e}\phi} - 4\frac{\partial}{\partial s}s\mu_{\mathrm{e}}u_{\mathrm{e}\phi}\frac{\partial n_{\mathrm{e}}}{\partial s}\right)$$
$$+ \frac{e}{m_{\mathrm{e}}}n_{\mathrm{e}}\dot{A}_{\phi} + 2\frac{e}{m_{\mathrm{e}}}\frac{\partial A_{\phi}}{\partial s}rn_{\mathrm{e}}u_{\mathrm{er}} - \nu_{\mathrm{ei3}}n_{\mathrm{e}}(u_{\mathrm{e}\phi} - u_{\mathrm{i}\phi}) + 2\frac{\partial A_{\phi}}{\partial s}\frac{\nu_{\mathrm{ei2}}}{B_{\theta}}rn_{\mathrm{e}}(u_{\mathrm{e}\theta} - u_{\mathrm{i}\theta})$$
$$+ 2\frac{m_{\mathrm{b}}}{m_{\mathrm{e}}}\frac{\partial A_{\phi}}{\partial s}\frac{\nu_{\mathrm{be2}}}{B_{\theta}}rn_{\mathrm{e}}(u_{\mathrm{e}\theta} - u_{\mathrm{i}\theta}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{e}}}\nu_{\mathrm{be3}}n_{\mathrm{b}}(u_{\mathrm{e}\phi} - u_{\mathrm{b}\phi})$$
$$- 2\frac{e^2B_{\phi}D_{\mathrm{e}}}{m_{\mathrm{e}}T_{\mathrm{e}}}\frac{\partial A_{\phi}}{\partial s}\left[rn_{\mathrm{e}}u_{\mathrm{e}\theta} + sn_{\mathrm{e}}\frac{B}{B_{\phi}}\left\langle\frac{\omega}{m}\right\rangle_{\mathrm{e}} + 2s\frac{1-\alpha_{\mathrm{e}}}{eB_{\phi}}\left(\frac{\partial}{\partial s}n_{\mathrm{e}}T_{\mathrm{e}} - T_{\mathrm{e}}\frac{\partial}{\partial s}n_{\mathrm{e}}\right)\right.$$
$$\left. - 2sn_{\mathrm{e}}\frac{1}{B_{\phi}}\frac{\partial}{\partial s}\phi\right] - \frac{e^2B_{\phi}^2D_{\mathrm{e}}}{m_{\mathrm{e}}T_{\mathrm{e}}}\left(\frac{B_{\theta}}{B_{\phi}}\right)^2 n_{\mathrm{e}}u_{\mathrm{e}\phi} - 2\nu_{\mathrm{L}}n_{\mathrm{e}}u_{\mathrm{e}\phi} - \nu_{0\mathrm{e}}n_{\mathrm{e}}u_{\mathrm{e}\phi} \tag{3.9}$$

$$\frac{\partial}{\partial t}\frac{3}{2}n_{\mathrm{e}}T_{\mathrm{e}} = -5\frac{\partial}{\partial s}ru_{\mathrm{er}}n_{\mathrm{e}}T_{\mathrm{e}} + \left(4\frac{\partial}{\partial s}s\chi_{\mathrm{e}}\frac{\partial}{\partial s}n_{\mathrm{e}}T_{\mathrm{e}} - 4\frac{\partial}{\partial s}s\chi_{\mathrm{e}}T_{\mathrm{e}}\frac{\partial n_{\mathrm{e}}}{\partial s}\right) + 2ru_{\mathrm{er}}\frac{\partial}{\partial s}n_{\mathrm{e}}T_{\mathrm{e}}$$
$$- e\frac{E_{\theta}}{r}rn_{\mathrm{e}}u_{\mathrm{e}\theta} - eE_{\phi}n_{\mathrm{e}}u_{\mathrm{e}\phi} - \frac{3}{2}\nu_{\mathrm{Tei}}n_{\mathrm{e}}(T_{\mathrm{e}} - T_{\mathrm{i}})$$
$$- \nu_{\mathrm{L}}T_{\mathrm{e}}(n_{\mathrm{e}} - n_{\mathrm{ediv}}) - \frac{3}{2}\nu_{\mathrm{L}T_{\mathrm{e}}}n_{\mathrm{e}}(T_{\mathrm{e}} - T_{\mathrm{ediv}})$$
$$- E_{\mathrm{H}}\nu_{\mathrm{ionize}}\frac{n_{\mathrm{e}}}{n_{01}+n_{02}+n_{03}}(n_{01}+n_{02}+n_{03}) + f_{\mathrm{ei}}P_{\mathrm{b}} + P_{\alpha\mathrm{e}} + P_{\mathrm{RFe}} - P_{\mathrm{Br}} \tag{3.10}$$

$$\frac{\partial}{\partial t}n_{\mathrm{i}} = -2\frac{\partial}{\partial s}rn_{\mathrm{i}}u_{\mathrm{ir}} + \frac{\nu_{\mathrm{ionize}}}{Z_{\mathrm{i}}}\frac{n_{\mathrm{e}}}{n_{01}+n_{02}+n_{03}}(n_{01}+n_{02}+n_{03}) - \frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}}(n_{\mathrm{e}} - n_{\mathrm{ediv}})$$
$$+ \nu_{\mathrm{b}}n_{\mathrm{b}} + \nu_{\mathrm{b}}gn_{\mathrm{b}}^{\mathrm{rp}} - f_{\mathrm{CX}}\frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} + \nu_{\mathrm{bL}}n_{\mathrm{b}} + S_{\mathrm{LC}} - \nu_{\mathrm{OL}}n_{\mathrm{i}} \tag{3.11}$$

$$\frac{\partial}{\partial t}rn_{\mathrm{i}}u_{\mathrm{ir}} = -2r\frac{\partial}{\partial s}u_{\mathrm{ir}}rn_{\mathrm{i}}u_{\mathrm{ir}} + \frac{u_{\mathrm{e}\theta}}{r}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - 2s\frac{\partial}{\partial s}\frac{n_{\mathrm{i}}T_{\mathrm{i}}}{m_{\mathrm{i}}}$$
$$- 2Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}sn_{\mathrm{i}}\frac{\partial}{\partial s}\phi + Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}B_{\phi}rn_{\mathrm{i}}u_{\mathrm{i}\theta} + 2Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}s\frac{\partial A_{\phi}}{\partial s}n_{\mathrm{i}}u_{\mathrm{i}\phi} \tag{3.12}$$

$$\frac{\partial}{\partial t}rn_{\mathrm{i}}u_{\mathrm{i}\theta} = -2\frac{\partial}{\partial s}ru_{\mathrm{ir}}rn_{\mathrm{i}}u_{\mathrm{i}\theta} + \left(4\frac{\partial}{\partial s}s\mu_{\mathrm{i}}\frac{\partial}{\partial s}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - 4\frac{\partial}{\partial s}s\mu_{\mathrm{i}}ru_{\mathrm{i}\theta}\frac{\partial n_{\mathrm{i}}}{\partial s} - 4\frac{\partial}{\partial s}\mu_{\mathrm{i}}rn_{\mathrm{i}}u_{\mathrm{i}\theta}\right)$$
$$- Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}n_{\mathrm{i}}r\dot{A}_{\theta} - Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}B_{\phi}rn_{\mathrm{i}}u_{\mathrm{ir}} - \nu_{\mathrm{NCi}}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\nu_{\mathrm{ei1}}rn_{\mathrm{e}}(u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta})$$
$$+ 2\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}s\frac{\partial A_{\phi}}{\partial s}\frac{\nu_{\mathrm{ei2}}}{B_{\theta}}n_{\mathrm{e}}(u_{\mathrm{i}\phi} - u_{\mathrm{e}\phi}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{i}}}\nu_{\mathrm{bi}}rn_{\mathrm{b}}(u_{\mathrm{i}\theta} - u_{\mathrm{b}\theta})$$
$$+ \frac{e^2B_{\phi}^2D_{\mathrm{e}}}{m_{\mathrm{i}}T_{\mathrm{e}}}\left[rn_{\mathrm{e}}u_{\mathrm{e}\theta} + 2\frac{\partial A_{\phi}}{\partial s}\frac{s}{B_{\phi}}n_{\mathrm{e}}u_{\mathrm{e}\phi} + sn_{\mathrm{e}}\frac{B}{B_{\phi}}\left\langle\frac{\omega}{m}\right\rangle_{\mathrm{e}} + 2s\frac{1-\alpha_{\mathrm{e}}}{eB_{\phi}}\left(\frac{\partial}{\partial s}n_{\mathrm{e}}T_{\mathrm{e}} - T_{\mathrm{e}}\frac{\partial}{\partial s}n_{\mathrm{e}}\right)\right.$$
$$\left. - 2sn_{\mathrm{e}}\frac{1}{B_{\phi}}\frac{\partial}{\partial s}\phi\right] - 2\nu_{\mathrm{L}}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - \nu_{0\mathrm{i}}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - \nu_{\mathrm{CX}}rn_{\mathrm{i}}u_{\mathrm{i}\theta}$$

$$+ rS_{\mathrm{LC}\theta} - \nu_{\mathrm{OL}} r n_{\mathrm{i}} u_{\mathrm{i}\theta} + F_{\mathrm{i}\theta}^{\mathrm{neo}} + T_{\mathrm{i}\theta} \tag{3.13}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_{\mathrm{i}} u_{\mathrm{i}\phi} = & -2\frac{\partial}{\partial s} r u_{\mathrm{ir}} n_{\mathrm{i}} u_{\mathrm{i}\phi} + \left(4\frac{\partial}{\partial s} s\mu_{\mathrm{i}} \frac{\partial}{\partial s} n_{\mathrm{i}} u_{\mathrm{i}\phi} - 4\frac{\partial}{\partial s} s\mu_{\mathrm{i}} u_{\mathrm{i}\phi} \frac{\partial n_{\mathrm{i}}}{\partial s}\right) - Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} n_{\mathrm{i}} \dot{A}_{\phi} - 2Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} \frac{\partial A_{\phi}}{\partial s} r n_{\mathrm{i}} u_{\mathrm{ir}} \\ & - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} \nu_{\mathrm{ei3}} n_{\mathrm{e}} (u_{\mathrm{i}\phi} - u_{\mathrm{e}\phi}) + 2\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} \frac{\partial A_{\phi}}{\partial s} \frac{\nu_{\mathrm{ei2}}}{B_{\theta}} r n_{\mathrm{e}} (u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{i}}} \nu_{\mathrm{bi}} n_{\mathrm{b}} (u_{\mathrm{i}\phi} - u_{\mathrm{b}\phi}) \\ & + 2\frac{e^2 B_{\phi} D_{\mathrm{e}}}{m_{\mathrm{i}} T_{\mathrm{e}}} \frac{\partial A_{\phi}}{\partial s} \left[r n_{\mathrm{e}} u_{\mathrm{e}\theta} + s n_{\mathrm{e}} \frac{B}{B_{\phi}} \left\langle \frac{\omega}{m} \right\rangle_{\mathrm{e}} + 2s \frac{1-\alpha_{\mathrm{e}}}{e B_{\phi}} \left(\frac{\partial}{\partial s} n_{\mathrm{e}} T_{\mathrm{e}} - T_{\mathrm{e}} \frac{\partial}{\partial s} n_{\mathrm{e}} \right) \right. \\ & \left. - 2s n_{\mathrm{e}} \frac{1}{B_{\phi}} \frac{\partial}{\partial s} \phi \right] + \frac{e^2 B_{\phi}^2 D_{\mathrm{e}}}{m_{\mathrm{i}} T_{\mathrm{e}}} \left(\frac{B_{\theta}}{B_{\phi}} \right)^2 n_{\mathrm{e}} u_{\mathrm{e}\phi} - 2\nu_{\mathrm{L}} n_{\mathrm{i}} u_{\mathrm{i}\phi} - \nu_{0\mathrm{i}} n_{\mathrm{i}} u_{\mathrm{i}\phi} - \nu_{\mathrm{CX}} n_{\mathrm{i}} u_{\mathrm{i}\phi} \\ & + S_{\mathrm{LC}\phi} - \nu_{\mathrm{OL}} n_{\mathrm{i}} u_{\mathrm{i}\phi} + T_{\phi,\mathrm{i}}\end{aligned} \tag{3.14}$$

$$\begin{aligned}\frac{\partial}{\partial t} \frac{3}{2} n_{\mathrm{i}} T_{\mathrm{i}} = & -5\frac{\partial}{\partial s} r u_{\mathrm{er}} n_{\mathrm{e}} T_{\mathrm{e}} + \left(4\frac{\partial}{\partial s} s\chi_{\mathrm{e}} \frac{\partial}{\partial s} n_{\mathrm{i}} T_{\mathrm{i}} - 4\frac{\partial}{\partial s} s\chi_{\mathrm{i}} T_{\mathrm{i}} \frac{\partial n_{\mathrm{i}}}{\partial s}\right) + 2r u_{\mathrm{ir}} \frac{\partial}{\partial s} n_{\mathrm{i}} T_{\mathrm{i}} \\ & + Z_{\mathrm{i}} e \frac{E_{\theta}}{r} r n_{\mathrm{i}} u_{\mathrm{i}\theta} + Z_{\mathrm{i}} e E_{\phi} n_{\mathrm{i}} u_{\mathrm{i}\phi} - \frac{3}{2} \nu_{\mathrm{Tei}} n_{\mathrm{e}} (T_{\mathrm{i}} - T_{\mathrm{e}}) + m_{\mathrm{b}} v_{\mathrm{b}} \frac{B_{\theta} u_{\mathrm{i}\theta} + B_{\phi} u_{\mathrm{i}\phi}}{B} \frac{P_{\mathrm{b}\parallel}}{E_{\mathrm{b}}} \\ & - \frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}} T_{\mathrm{i}} (n_{\mathrm{e}} - n_{\mathrm{ediv}}) - \frac{3}{2} \nu_{\mathrm{LT_i}} n_{\mathrm{i}} (T_{\mathrm{i}} - T_{\mathrm{idiv}}) \\ & + \frac{3}{2} \frac{\nu_{\mathrm{ionize}}}{Z_{\mathrm{i}}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} (n_{01} T_{01} + n_{02} T_{02} + n_{03} T_{03}) - \frac{3}{2} \nu_{\mathrm{CX}} n_{\mathrm{i}} (T_{\mathrm{i}} - T_{01}) \\ & + (1 - f_{\mathrm{ei}}) P_{\mathrm{b}} + P_{\alpha\mathrm{i}} + P_{\mathrm{RFi}}\end{aligned} \tag{3.15}$$

$$\frac{\partial}{\partial t} n_{\mathrm{b}} = \frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} - g\frac{P_{\mathrm{b}\perp}}{E_{\mathrm{b}}} - \nu_{\mathrm{b}} n_{\mathrm{b}} - \nu_{\mathrm{bL}} n_{\mathrm{b}} - \nu_{\mathrm{eff}} g(\sqrt{\delta} n_{\mathrm{b}} - n_{\mathrm{b}}^{\mathrm{rp}}) + 4\frac{\partial}{\partial s} s f_t D_{\mathrm{b}} \frac{\partial}{\partial s} n_{\mathrm{b}} \tag{3.16}$$

$$\begin{aligned}\frac{\partial}{\partial t} r n_{\mathrm{b}} u_{\mathrm{b}\theta} = & -Z_{\mathrm{b}} \frac{e}{m_{\mathrm{b}}} n_{\mathrm{b}} r \dot{A}_{\theta} - \nu_{\mathrm{be1}} r n_{\mathrm{b}} (u_{\mathrm{b}\theta} - u_{\mathrm{e}\theta}) + 2s \frac{\partial A_{\phi}}{\partial s} \frac{\nu_{\mathrm{be2}}}{B_{\theta}} n_{\mathrm{b}} (u_{\mathrm{b}\phi} - u_{\mathrm{e}\phi}) - \nu_{\mathrm{bi}} r n_{\mathrm{b}} (u_{\mathrm{b}\theta} - u_{\mathrm{i}\theta}) \\ & - \nu_{0\mathrm{b}} r n_{\mathrm{b}} u_{\mathrm{b}\theta} - \nu_{\mathrm{CX}} r n_{\mathrm{b}} u_{\mathrm{b}\theta} + r v_{\mathrm{b}\theta} \frac{P_{\mathrm{b}\parallel}}{E_{\mathrm{b}}} - \nu_{\mathrm{bL}} r n_{\mathrm{b}} u_{\mathrm{b}\theta} - \nu_{\mathrm{eff}} g \sqrt{\delta} r n_{\mathrm{b}} u_{\mathrm{b}\theta} \\ & + 4\frac{\partial}{\partial s} s f_t D_{\mathrm{b}} \frac{\partial}{\partial s} r n_{\mathrm{b}} u_{\mathrm{b}\theta} - 4\frac{\partial}{\partial s} \left(s f_t D_{\mathrm{b}} \frac{\partial r u_{\mathrm{b}\theta}}{\partial s} \right) n_{\mathrm{b}}\end{aligned} \tag{3.17}$$

$$\begin{aligned}\frac{\partial}{\partial t} n_{\mathrm{b}} u_{\mathrm{b}\phi} = & -Z_{\mathrm{b}} \frac{e}{m_{\mathrm{b}}} n_{\mathrm{b}} \dot{A}_{\phi} - \nu_{\mathrm{be3}} n_{\mathrm{b}} (u_{\mathrm{b}\phi} - u_{\mathrm{e}\phi}) + 2\frac{\partial A_{\phi}}{\partial s} \frac{\nu_{\mathrm{be2}}}{B_{\theta}} n_{\mathrm{b}} (u_{\mathrm{b}\theta} - u_{\mathrm{e}\theta}) - \nu_{\mathrm{bi}} n_{\mathrm{b}} (u_{\mathrm{b}\phi} - u_{\mathrm{i}\phi}) \\ & - \nu_{0\mathrm{b}} n_{\mathrm{b}} u_{\mathrm{b}\phi} - \nu_{\mathrm{CX}} n_{\mathrm{b}} u_{\mathrm{b}\phi} + v_{\mathrm{b}\phi} \frac{P_{\mathrm{b}\parallel}}{E_{\mathrm{b}}} - \nu_{\mathrm{bL}} n_{\mathrm{b}} u_{\mathrm{b}\phi} - \nu_{\mathrm{eff}} g \sqrt{\delta} n_{\mathrm{b}} u_{\mathrm{b}\phi} \\ & + 4\frac{\partial}{\partial s} s f_t D_{\mathrm{b}} \frac{\partial}{\partial s} n_{\mathrm{b}} u_{\mathrm{b}\phi} - 4\frac{\partial}{\partial s} \left(s f_t D_{\mathrm{b}} \frac{\partial u_{\mathrm{b}\phi}}{\partial s} \right) n_{\mathrm{b}}\end{aligned} \tag{3.18}$$

$$\frac{\partial}{\partial t} n_{01} = 4\frac{\partial}{\partial s} s D_{01} \frac{\partial}{\partial s} n_{01} - \frac{1}{Z_{\mathrm{i}}} \nu_{\mathrm{ionize}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} n_{01} - \nu_{\mathrm{CX}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02}} n_{01}$$

$$+\gamma_0\frac{\nu_{\rm L}}{Z_{\rm i}}\left(n_{\rm e}-n_{\rm ediv}\right)+2\frac{\partial}{\partial s}r\Gamma^{\rm puff}\Big|_{\rm b} \tag{3.19}$$

$$\frac{\partial}{\partial t}n_{02}=4\frac{\partial}{\partial s}sD_{02}\frac{\partial}{\partial s}n_{02}-\frac{1}{Z_{\rm i}}\nu_{\rm ionize}\frac{n_{\rm e}}{n_{01}+n_{02}+n_{03}}n_{02}+\nu_{\rm CX}\frac{n_{\rm i}}{n_{01}+n_{02}}n_{01} \tag{3.20}$$

$$\frac{\partial}{\partial t}n_{03}=4\frac{\partial}{\partial s}sD_{03}\frac{\partial}{\partial s}n_{03}-\frac{1}{Z_{\rm i}}\nu_{\rm ionize}\frac{n_{\rm e}}{n_{01}+n_{02}+n_{03}}n_{03}+f_{\rm CX}\frac{P_{\rm b}}{E_{\rm b}} \tag{3.21}$$

$$\frac{\partial}{\partial t}n_{\rm b}^{\rm rp}=\frac{P_{\rm b\perp}}{E_{\rm b}}+\nu_{\rm eff}(\sqrt{\delta}n_{\rm b}-n_{\rm b}^{\rm rp})-\nu_{\rm b}n_{\rm b}^{\rm rp}-2\frac{\partial}{\partial s}ru_{\rm b}^{\rm rp}n_{\rm b}^{\rm rp}+4\frac{\partial}{\partial s}sD_{\rm b}^{\rm rp}\frac{\partial}{\partial s}n_{\rm b}^{\rm rp} \tag{3.22}$$

4 Boundary Conditions

num.	name	variable	center	edge
1	LQm1	ϕ	N	0
2	LQm2	$r\dot{A}_\theta$	0	$2s_bB_{\rm V}$
3	LQm3	$\dot{A}_\phi$	N	$-2bB_{\theta{\rm b}}$
4	LQm4	A_ϕ	×	×
5	LQm5	rA_θ	×	×
6	LQe1	$n_{\rm e}$	N	N(×)
7	LQe2	$rn_{\rm e}u_{{\rm e}r}$	0	×(0)
8	LQe3	$rn_{\rm e}u_{{\rm e}\theta}$	0	0
9	LQe4	$n_{\rm e}u_{{\rm e}\phi}$	N	0
10	LQe5	$n_{\rm e}T_{\rm e}$	N	N
11	LQi1	$n_{\rm i}$	N	N(×)
12	LQi2	$rn_{\rm i}u_{{\rm i}r}$	0	×(0)
13	LQi3	$rn_{\rm i}u_{{\rm i}\theta}$	0	0
14	LQi4	$n_{\rm i}u_{{\rm i}\phi}$	N	0
15	LQi5	$n_{\rm i}T_{\rm i}$	N	N
16	LQb1	$n_{\rm b}$	×(N)	×(N)
17	LQb3	$rn_{\rm b}u_{{\rm b}\theta}$	×(0)	×(0)
18	LQb4	$n_{\rm b}u_{{\rm b}\phi}$	×(N)	×(0)
19	LQn1	n_{01}	N	×($2bS_{\rm gas}$)
20	LQn2	n_{02}	N	0
20	LQn3	n_{03}	N	0
21	LQr1	$n_{\rm b}^{\rm rp}$	×(N)	×(N)

0: Dirichlet condition ($u=0$)

N: Neumann condition ($u'=0$)

value: Neumann condition ($u'=$ value)

×: no condition

Excess BCs are imposed on the equations LQe1 and LQi1, from the aspect of the order of their derivative. However, these BCs must be required to keep quasi-neutrality.